Ministry of Education and Science of the Russian Federation

National Research University – Novosibirsk State University (NRU NSU)

Department of Mechanics and Mathematics

MASTER EDUCATIONAL PROGRAMME

FOR TEACHING FOREIGN STUDENTS IN ENGLISH

**«Modern Trends in Discrete Mathematics**

**and Combinatorial Optimization»**

qualification (degree) of the graduate

Master of Mathematics and Computer Science

Full-time tuition

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«Routing and Scheduling»

«English Academic Writing»

«History of Numerical Statistical Modeling and Simulation»

«Applied Statistics»

«Financial Mathematics»

«Number Theory»

«Methods of Discrete Simulation»

«Numerical Methods»

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**Introduction**

**National Research University – Novosibirsk State University.**

**Department of Mechanics and Mathematics**

Novosibirsk State University (NSU; see the site <http://www.nsu.ru/exp/index.jz?lang=en>) was established in 1958. NSU was growing up together with world known Novosibirsk Scientific Center (Akademgorodok) focusing on training highly qualified specialists for science and education.

The Department of Mechanics and Mathematics (DMM; see the site <http://www.nsu.ru/exp/en/education/mechanics_and_mathematics>) is one of the leading divisions of NSU. It was founded in 1961. The DMM has three levels of education: for Bachelor degree, for Master degree and postgraduate study. Students get trained in the following fields: Mathematics, Mathematics and Computer Science, Applied Mathematics and Information Science, Mechanics and Mathematical Simulation.

The main courses on DMM are held by outstanding scientists from institutes of Siberian Branch (SB) of Russian Academy of Sciences (RAS). Apart from attending the courses students have an opportunity to join leading scientific schools in these institutes. Both professors and students take an active part in corporative projects with researchers from leading universities and scientific organizations of Russia and other countries. Working in RAS research institutes senior students carry out research in modern fields of mathematics and its applications (including various fundamental and actual issues of mathematical modeling and computer science). Such specialization diversity allows students to prepare both for carrying out research in scientific institutes and for practical work in many areas applying the latest information technologies.

Every year the International student scientific conference «Students and the progress in science and technologies» takes place in NSU (see the site <http://issc.nsu.ru/index.php?lang=1>), where students and young scientists can deliver talks on their researches and get acquainted with the results of their colleagues.

In 2009, after the complete selection among the leading Russian universities, the Novosibirsk State University got the category National Research University (NRU). This category was awarded by the Government of the Russian Federation on the basis of special Development Program of NSU for the period 2009-2018 (see the site <http://www.nsu.ru/exp/en/university/national_research_university>). The strategy goal of the Program is (quote): *“to form a research-educational system of national and global importance…which will be able to provide an advised specialist training based on science, education and business integration”.*

In 2013 NRU NSU became one of 15 Russian universities that had won in the competition the right to get a subsidy that will favour the university advancement in the world ratings.

**Chair of Theoretical Cybernetics DMM NRU NSU**

The Chair of Theoretical Cybernetics (see the site <http://tc.nsu.ru/> [in Russian]) was founded in 1965 by A.A. Lyapunov, a well-known mathematician, the corresponding member of the USSR Academy of Sciences, who was the head of the chair till 1973. Later on, the chair was headed by A.P. Ershov, the corresponding member of the USSR Academy of Sciences, V.L. Makarov, the corresponding member of the USSR Academy of Sciences and Professor V.T. Dementyev. Since 2010 Professor A.I. Erzin has been the head of the Chair.

Every year, about 30 students of the Department major at the Chair of Theoretical Cybernetics in the following areas:

Operations research

Optimization methods

Discrete analysis

Graph theory

Scheduling theory

Coding theory

Optimal control

Data analysis and automatic recognition.

The main research fields include proving structural properties of discrete objects, analyzing the data structures, finding out the complexity status of optimization problems, developing precise or approximate algorithms for their solution, getting the performance bounds (apriori or aposteriori) for the algorithms, etc.

Almost all professors and other staff of the Chair work as scientific researchers of Russian Academy of Sciences (RAS). The main institution for the Chair is Sobolev Institute of Mathematics RAS. Most members of the staff have scientific degrees. This makes a good environment for high quality education and provides opportunities for students participating in actual scientific projects and their industrial applications. For students with keen interest in discrete mathematics and combinatorial optimization the Chair offers a wide range of advanced professional optional courses.

Every three years the Chair organizes an international conference «Discrete Optimization and Operations Research» (DOOR) where the leading specialists in various fields of discrete mathematics and combinatorial optimization deliver their talks. In 2013 the first young scientists’ school «Discrete Models and Decision Making Methods» took place just before the DOOR conference.

**Development of the Master Educational Programme (MEP)**

**«Modern trends in discrete mathematics and combinatorial optimization»**

The Development Program of NRU NSU for the period 2009-2018 implies also the introduction of individual educational pathways on the basis of the University Master Training Center and organization of English-speaking training groups (more detailed information can be found at the site <http://www.nsu.ru/exp/en/university/national_research_university>). A certain contribution to development of such Center is elaboration of the Master Educational Programme (MEP) «Modern trends in discrete mathematics and combinatorial optimization» for teaching foreign students in English (author – professor the Chair of Theoretical Cybernetics NRU NSU A.V.Pyatkin). This MEP allows student to get Russian state diploma at the end of education because it complies the State Master Standard (SMS) for specialty 010200 – «Mathematics and Computer Science» (approved by the Order № 760 of Ministry of Education and Science of the Russian Federation on December, 21; 2009; details [in Russian] can be found at the site <http://www.referent.ru/1/150164>).

During the last century the discrete and computational mathematics got boosting development which is connected mainly with the widespread involvement of computers and computer technologies into the everyday life. This required the development of new methods of studying, processing, transmitting, storing, protecting and analyzing the large volumes of information and also training the specialists knowing all these methods. A modern specialist in discrete mathematics and combinatorial optimization has to possess a deep knowledge of graph theory, scheduling theory, coding theory, cryptography, operations research, data analysis methods, mathematical modeling, linear, convex and integer programming, complexity theory, algorithms development and analysis, etc. The staff of the Chair of Theoretical Cybernetics includes world level specialists in all these fields. They carry out active research and teaching work, regularly take part in international scientific conferences and therefore they always know the most recent trends and methods in their research areas. Since most of these specialists are also good English speakers, the development of MEP «Modern trends in discrete mathematics and combinatorial optimization» for teaching foreign students in English at the Chair of Theoretical Cybernetics looks rational and actual.

**Information about the author – prof. A.V.Pyatkin**

**Scientific biography**

Pyatkin Artem Valeryevich was born 14th of November 1973 in Novosibirsk, USSR. In 1996 he graduated the Master degree at Department of Mathematics and Mechanics of Novosibirsk State University. In 1999 he got his PhD in Sobolev Institute of Mathematics and began to work there as researcher, senior researcher (since 2001) and leading researcher (since 2012). He got twice postdoc positions abroad: in Bergen University (Norway, 2004-2006) and in Durham University (UK, 2009-2012). In 2009 he defended his Doctor dissertation “Incidentor coloring and other problems on graphs: the algorithmic aspect”.

A.V. Pyatkin is an active participant of Russian and international scientific projects. In 2012-2013 he was the supervisor of the RFBR project 12-01-33028 «Optimization problems on discrete structures». From 1995 till 2013 he published more than 100 scientific papers, among them 58 journal papers. The research interests of A.V. Pyatkin include graph theory, algorithms and complexity, approximation algorithms, data analysis problems.

Since 2003 he delivers lectures in NSU (the course «Optimization Methods» for the third year students of DMM NSU). Seven NSU students got their diplomas under his supervision.

In 2013 A.V. Pyatkin was the chief of organizing committee of the first all-Russia young scientists’ school «Discrete Models and Decision Making Methods» (see the site <http://math.nsc.ru/LBRT/k4/school2013/> [in Russian]).

**Last 5 years journal papers and teaching books**

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9. Kel’manov A. V., Pyatkin A. V. On complexity of some cluster analysis problems of vector sequences // **Discrete Analysis and Operations Research.** 2013. V. 20, № 2. P. 47-57 (in Russian).

10. Kitaev S.V., Pyatkin A.V. On avoidance of V- and Λ-patterns in permutations // **Ars Combinatoria,** 2010. Vol. 97. P. 203-215.

11. Larin R.M., Plyasunov A.V., Pyatkin A.V. **Optimization methods. Examples and exercises.** NSU Publ., Novosibirsk, 2009 (in Russian). 138 pages.

12. Pyatkin A.V. Triangle-free 2*P*3-free graphs are 4-colorable // **Discrete Mathematics.** 2013. V. 313. P. 715–720.

13. Pyatkin A.V., Chernykh I.D. The Open Shop Problem with Routing at a Two-Node Network and Allowed Preemption **// Journal of Applied and Industrial Mathematics.** 2012. Vol. 6, No. 3. P. 346–354.

14. Vernitski A., Pyatkin A.V. Astral graphs (threshold graphs), scale-free graphs and related algorithmic questions // **Journal of Discrete Algorithms**, 2012, V.12, P.24–28.

**Brief description of the MEP «Modern Trends in**

**Discrete Mathematics and Combinatorial Optimization»**

**Definition of the Master Educational Programme (MEP)**

The **master educational programme (MEP)** **«Modern Trends in Discrete Mathematics and Combinatorial Optimization»** is a system of educational documents based the State Master Standard (SMS) for specialty 010200 – «Mathematics and Computer Science» (approved by the Order № 760 of Ministry of Education and Science of the Russian Federation on December, 21; 2009) and the requirements for graduates of magistracy of Department of Mechanics and Mathematics (DMM) of National Research University – Novosibirsk State University (NRU NSU). The MEP includes curriculum, programs of courses, conditions for admission to the MEP, and acquired knowledge and skills of graduates.

**Aims and objectives of the MEP**

The purpose of the MEP «Modern Trends in Discrete Mathematics and Combinatorial Optimization» is training highly qualified specialists in the field of discrete mathematics and combinatorial optimization, capable of working in research institutes, universities, programming firms, engineering companies, etc. The training is based on requirements of the SMS 010200 and NRU NSU.

**Training period of the MEP**

The training period of the MEP «Modern Trends in Discrete Mathematics and Combinatorial Optimization» is equal to 2 years: 4 semesters, 4 test sessions, 4 exam sessions (see below the section «Approximate curriculum for training of masters according to the MEP»).

An academic year begins at September, 1. The “autumn” semesters (the first and the third semesters of the MEP) proceed 17 weeks: from September, 1 till the end of December. The “winter” test sessions (after the first and the third semesters of the MEP) occur at the end of December. The “winter” examination sessions proceed in January.

The “winter” holydays (after the first and the third semesters of the MEP) proceed one week at the beginning of February.

The second “spring” semester proceeds 17 weeks: from February till the end of May. The second “summer” test and examination sessions proceed in June. The “summer” holydays (after the second semester of the MEP) proceeds two months: from July, 1 till August, 31.

The fourth “spring” semester proceeds 14 weeks: from February till May. The fourth “summer” test and examination sessions proceed at the end of May. Then (in June) the procedures of final state validation (including the defense of the master dissertation) take place.

**Training capacity of the MEP**

Training capacity for study within the MEP «Modern Trends in Discrete Mathematics and Combinatorial Optimization», including classroom work, independent and research student’s work and the time taken for quality control, is equal to 120 credits (units of study); one credit (unit) is equal to 36 academic hours – see below the section «Approximate curriculum for training of masters according to the MEP».

**Conditions for admission to the MEP**

**«Modern Trends in Discrete Mathematics and Combinatorial Optimization»**

**Educational level of the entrant**

The entrant to the MEP should have a high educational diploma (certificate) on Bachelor program in the field of mathematics, natural science (physics, chemistry, biology, geology, etc.) or economics. This program must include an advanced mathematical course (courses) with basic elements of calculus, functional analysis, algebra, mathematical logic, probability theory and mathematical statistics, numerical mathematics, and programming. Desirable knowledge of English language: TOEFL Internet based equivalent score 50-70 or IELTS 4.5-6 score (intermediate or upper intermediate). Some knowledge of Russian language (for scientific and household communication) is also useful (but not necessary).

**Selection procedure to the MEP (the major dates)**

Till ***May*, 15** the applicant send the following copies of documents by electronic mail to the supervisor of the program Professor *Pyatkin Artem Valeryevich* (e-mail: [artem@math.nsc.ru](mailto:artem@math.nsc.ru)):

1. Diploma (or some analogous document) on programme of level of a bachelor degree or certificate (ordering) about passing at the moment such programme.

2. Recommendation of the professor of mathematics or applied mathematics.

3. Biography of the applicant.

4. Motivational letter (1-2 pages) on entrance to the MEP.

It is also recommended (but not obligatory) to send the following:

1. A text or description of the bachelor’s diploma work (it will help to assign a proper scientific supervisor).

2. A document confirming knowledge of English (TOEFL, IELTS or another international English test certificate). Note that studying the MEP requires a good knowledge of English, and it is responsibility of the student to evaluate whether his/her level of language skill is sufficient for studying the MEP.

Till ***June*, 1** the preliminary selection of applicants is made.

Till ***June*, 10** the interview (in internal form or by Skype) with committee of leading professors of the Chair of Numerical Mathematics NRU NSU is carried out. Features of this interview are reflected in the following section of the programme.

Till ***July*, 1** the admittance to the MEP occurs.

**Admission test and interview on the Chair of Theoretical Cybernetics NRU NSU**

The entrant to the MEP should pass the admission test in the form of interview with a committee of leading professors of the Chair of Theoretical Cybernetics NRU NSU (approximate date: May or June). This interview includes control questions and simple test tasks on the basics of calculus, functional analysis, algebra, mathematical logic, probability theory and mathematical statistics, numerical mathematics, and programming. Examples of admission interview control questions and tests are presented below in the corresponding section of the MEP. Also during the interview the entrant can express his own training goals and preferred specialization within the MEP and indicate a desired scientific supervisor (if he already has chosen one).

If the committee makes the decision on acceptance of the applicant, the entrant’s scientific supervisor (one of the professors of the Chair of Theoretical Cybernetics NRU NSU) is assigned. This person defines the topic of scientific research for preparation of term and dissertations works. The committee can also determine an individual additional leveling educational list of basic courses (in the case when the applicant shows insufficiently profound knowledge during the interview).

The interview can be realized in person or by Skype. Preferable term of carrying out interview is May or June of current year. Concerning the organization of interview address to Professor *Pyatkin Artem Valeryevich*; e-mail: [artem@math.nsc.ru](mailto:artem@math.nsc.ru); telephone: +7-383-3634546.

**Formal reception to the MEP. Training payment**

In case of successful passing of interview, the testing committee of the Chair of Theoretical Cybernetics NRU NSU formulates the written recommendation for the entrant to the MEP (this document must be signed by the Head and Secretary of the Chair and also by the nominated scientific supervisor of the entrant) and sends it to dean’s office of Department of Mechanics and Mathematics (DMM) NRU NSU.

The training for master on DMM NRU NSU is paid (in 2014-1015 the corresponding price is equal to 5200 US dollars for one year).

For further details on training do not hesitate to ask Professor *Pyatkin Artem Valeryevich*; e-mail: [artem@math.nsc.ru](mailto:artem@math.nsc.ru); telephone: +7-383-3634546.

**Learning outcomes of the MEP**

**«Modern Trends in Discrete Mathematics and Combinatorial Optimization»**

**Area of professional activities of MEP graduates**

The area of professional activities of MEP graduates corresponds to the SMS for specialty 010200 – «Mathematics and Computer Science» (details [in Russian] can be found at the site <http://www.referent.ru/1/150164>). It includes the following scientific and applied aspects of discrete mathematics and combinatorial optimization:

* Research activity in the areas using mathematical methods and computer science technologies (in particular, in the following areas: decision-making, discrete extremal problems, combinatorics, graph theory, scheduling, data mining, cryptography, coding, etc);
* Solving practical problems with the use of mathematical modeling and programming;
* Development of effective methods for solving problems of natural science, economics and administrating;
* Teaching of courses in mathematics and computer science.

A MEP graduate can work in the following organizations:

* Research institutes of mathematical or computer science profile;
* Colleges and universities;
* Analytical or engineering departments of large firms in various industries;
* Analytical or engineering departments of banks.

A MEP graduate can also continue his study as a PhD student of the Chair of Theoretical Cybernetics NRU NSU.

**Objects of professional activities of MEP graduates**

The objects of research scientific activity of the MEP graduates correspond to the SMS for specialty 010200 – «Mathematics and Computer Science».These objects include notions, conjectures, theorems and mathematical models of discrete mathematics and combinatorial optimization. In particular, discrete modeling, precise and approximate algorithms (including heuristics), optimization methods, structural theorems, complexity analysis techniques, data mining technologies, programming modules can be considered as such objects. Most of them can be applied for solving actual scientific, engineering, economic and programming problems.

**Types of professional activities of MEP graduates**

The MEP students are preparing for the following main types of professional activity:

- *scientific and research activities* (within the MEP, this training is provided by probation periods at the Chair of Theoretical Cybernetics NRU NSU and laboratories of Sobolev Institute of Mathematics – IM SB RAS);

- *project, production and technological activities* (within the MEP, this training is provided by the participation in execution of scientific projects in IM SB RAS);

*- social and personal improvement*.

Additional professional activities of the graduate may be:

- *educational activities;*

*- consulting and consortium activities;*

*- organizational and management activities;*

*- normative and methodological activities;*

*- socially-oriented activities.*

These types of professional activity correspond to the SMS for specialty 010200 – «Mathematics and Computer Science».

**Purposes of professional activities of MEP graduates**

A graduate of MEP «Modern Trends in Discrete Mathematics and Combinatorial Optimization» should be able to apply the obtained skills in the following professional activities:

*Scientific research activity:*

* Applying methods of mathematical and algorithmic modeling in studying actual processes and objects for finding effective solutions of wide range of scientific and applied problems;
* Analysis and generalization of modern research results and methods in the field of discrete mathematics and combinatorial optimization;
* Development of new mathematical models and algorithms on the base of theoretical results for solving various optimization problems;
* Research work on mathematics and computer science.

*Industrial activity:*

* Development of programs and mathematical support for computers;
* Use of the modern mathematical methods and computer programs for solving practical problems;
* Development of data protection methods and systems
* Data mining, processing and analysis with the use of modern methods;

*Organizational and administrative activity:*

* Organization of research teams work;
* Organization of scientific conferences and workshops;
* Making expertise of works in the field of mathematics and computer science;

*Educational activity:*

* Delivering lectures and teaching seminars on the main courses of discrete mathematics and combinatorial optimization;

These purposes of professional activity correspond to the SMS for specialty 010200 – «Mathematics and Computer Science».

**Acquired knowledge and skills (according to the MEP curriculum)**

According to the SMS for specialty 010200 – «Mathematics and Computer Science», the MEP «Modern Trends in Discrete Mathematics and Combinatorial Optimization» includes the following cycles and parts (see the next section «Approximate curriculum for training of masters according to the MEP»):

– *M.1 «General scientific cycle» (basic part and variable part);*

– *M.2 «Professional cycle» (basic part and variable part);*

– *M.3 «Scientific practice and research work»;*

– *M.4 «Final state validation».*

In **the section M.1 «General scientific cycle»** the competences **GCC-1-6,9,10** and  **PC-1,2,6,8,11,12,14,16** (see the previous section) are formed. According to the SMS for specialty 010200 – «Mathematics and Computer Science», the supposed results of mastering disciplines of this part of the MEP «Modern Trends in Discrete Mathematics and Combinatorial Optimization» should be the following. **The MEP graduate student**

**should know** modern concepts of science, the place of natural and humanitarian sciences in the development of the scientific worldview;

**should be able to** perform a conceptual analysis of information from various sources, know modern algorithms of applied mathematics and numerical methods;

**should possess** skills to formulate non-mathematical types of data (including humanitarian) in the form of mathematical problems by means of mathematical and algorithmical modeling.

In **the section M.2 «Professional cycle»** the competences **GCC-5,6** and  **PC-4,7,9,1,-13** are formed. As the result of the part’s study **the** **MEP graduate student**

**should know** fundamental concepts and professional results, system methodologies in the area of discrete mathematics and combinatorial optimization, the current state and the principal ideas of modern methods in this area;

**should be able to** get new knowledge and apply it in professional activity; use modern theories, methods, systems and means of discrete mathematics and combinatorial optimization for solving research and application problems.

**should possess** skills of program implementation of developed methods for solving practical problems.

In **the section** **M.3** «**Scientific practice and research work**» the competencies **GCC-1-10** and **PC-1,3** are formed.According to the SMS for specialty 010200 – «Mathematics and Computer Science», **the MEP graduate student must get the following skills:**

* The ability to carry out independent scientific research and obtain scientific results in the area of discrete mathematics and combinatorial optimization (proving structural theorems on the properties of discrete objects, development and analysis of algorithms for the problems of discrete mathematics and combinatorial optimization, data analysis and processing, etc);
* Use of modern software and hardware information technologies during scientific research;
* Ability to present scientific results in front of various audiences;
* Ability to work in a research team and in a company.

In **the section** **M.4** «**Final state validation**» **the MEP graduate student should be able to:**

* Use modern methods for research and solving of scientific and applied problems in the area of discrete mathematics and combinatorial optimization;
* Apply methods of mathematics and computer science for solving practical problems.

**Structure of** **the MEP**

**«Modern Trends in Discrete Mathematics and Combinatorial Optimization»**

**The main table**

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Discipline title | Credits | In total  hours | Class-  room  work | Indepen-  dent  work | 1 sem. | 2 sem. | 3 sem. | 4 sem. | Sertifica-  tion form |
| **M.1 General scientific cycle** | **26** | **1080** | **13** | **13** | **8** | **10** | **8** |  |  |
| **Basic part** | **12** | **432** | **6** | **6** | **4** | **4** | **4** |  |  |
| History of Numerical Statistical Modeling and Simulation | 2 | 72 | 1 | 1 |  |  | 2 |  | Test |
| English Academic Writing | 8 | 288 | 4 | 4 | 2 | 2 |  |  | Test |
| Number Theory | 2 | 72 | 1 | 1 |  |  | 2 |  | Exam |
| Applied Statistics | 2 | 72 | 1 | 1 | 2 |  |  |  | Test |
| Financial Mathematics | 2 | 72 | 1 | 1 |  | 2 |  |  | Exam |
| **Variable part** | **14** | **504** | **7** | **7** | **4** | **6** | **4** |  |  |
| Numerical Methods | 4 | 144 | 2 | 2 | 2 | 2 |  |  | Ex/test |
| Stochastic Processes | 4 | 144 | 2 | 2 | 2 | 2 |  |  | Ex/test |
| Random Walks | 2 | 72 | 1 | 1 |  | 2 |  |  | Exam |
| Modern Methods of Computational Mathematics | 2 | 72 | 1 | 1 | *2* |  | 2 |  | Exam |
| Numerical Modeling of Discrete Random Processes and Fields | 2 | 72 | 1 | 1 | 2 |  | *2* |  | Exam |
| Markov Chains | 2 | 72 | 1 | 1 |  | 2 |  |  | Exam |
| Methods of Discrete Simulation | 2 | 72 | 1 | 1 | 2 |  | *2* |  | Exam |
| Theory of Programming | 2 | 72 | 1 | 1 | *2* |  | 2 |  | Exam |
| **M.2 Professional cycle** | **32** | **1152** | **16** | **16** | **8** | **8** | **10** | **6** |  |
| **Basic part** | **16** | **576** | **8** | **8** | **4** | **4** | **6** | **2** |  |
| Combinatorial Optimization | 2 | 72 | 1 | 1 | 2 |  |  |  | Exam |
| Operations Research | 4 | 144 | 2 | 2 |  | 4 |  |  | Ex/test |
| Graph Theory | 2 | 72 | 1 | 1 | 2 |  |  |  | Exam |
| Scheduling Theory | 2 | 72 | 1 | 1 |  |  | 2 |  | Exam |
| Coding Theory | 2 | 72 | 1 | 1 |  |  | 2 |  | Exam |
| Cryptography and Cryptanalysis | 4 | 144 | 2 | 2 |  |  | 2 | 2 | Ex/test |
| **Variable part** (courses at the choice of the student) | **8** | **288** | **4** | **4** | **2** | **2** | **2** | **2** |  |
| **Scientific seminar** (at the choice of the student) | **8** | **288** | **4** | **4** | **2** | **2** | **2** | **2** | Test |
| *List of possible courses at the choice of the student:* |  |  |  |  |  |  |  |  |  |
| Integer Programming | 2 | 72 | 1 | 1 | *2* |  | 2 |  | Exam |
| Machine Learning and Data Mining | 2 | 72 | 1 | 1 | 2 |  | *2* |  | Exam |
| Mathematical Models in Logistics | 2 | 72 | 1 | 1 |  | *2* |  | 2 | Exam |
| Approximation Algorithms | 2 | 72 | 1 | 1 |  | *2* |  | 2 | Exam |
| Combinatorial Problems on Cayley Graphs | 2 | 72 | 1 | 1 | 2 |  | *2* |  | Exam |
| Combinatorial Designs | 2 | 72 | 1 | 1 | *2* |  | 2 |  | Exam |
| Routing and Scheduling | 2 | 72 | 1 | 1 |  | 2 |  | *2* | Exam |
| **M.3 Scientific practice and research work** | **50** | **1800** | **5** | **45** | **8** | **10** | **14** | **18** |  |
| Research work on the topic of master dissertation | 30 | 1080 | 5 | 25 | 8 | 8 | 8 | 6 | Test |
| Reports at scientific seminars and conferences | 4 | 144 |  | 4 |  | 2 |  | 2 |  |
| Development of the master dissertation | 16 | 576 |  | 16 |  |  | 6 | 10 |  |
| **M.4 Final state validation** | **12** | **432** |  | **12** |  |  |  | **12** |  |
| Passing the state exam | 6 | 144 | 2 | 4 |  |  |  | 6 | Grade |
| Preparation and defending of master dissertation | 6 | 144 |  | 6 |  |  |  | 6 | Grade |
| **TOTAL:** | **120** | **4320** | **34** | **86** | **24** | **28** | **32** | **36** |  |

Remarks.

1. Each semester has 18 weeks: *first and third semesters* – from September, 1 till December, 15; test session – from December, 16 till December, 30; exam session – from January, 8 till January, 31; *second and fourth semesters* – from February, 8 till May, 20; test session – from May, 21 till May, 31; exam session – from June, 1 till June, 20

2. All courses of the MEP «Modern Trends in Discrete Mathematics and Combinatorial Optimization» correspond to the specialty 010200 – «Mathematics and Computer Science»

3. For the courses of the Variable part (courses at the choice of the student) of M.1 General scientific cycle and M.2 Professional cycle all possible semesters for studying are presented in the curriculum. The recommended semesters for studying are indicated by the *italic* font.

**Semester programs of the MEP**

THE FIRST (“AUTUMN”) SEMESTER

***Obligatory courses of the section “General scientific cycle. Basic part”; total volume – 4 credits (units)***

*English Academic Writing* – the first part of the discipline; 2 credits (units)

*Applied Statistics* – full course; *2* credits (units)

***Obligatory courses of the section M.2 “Professional cycle. Basic part”; total volume – 4 credits (units)***

*Combinatorial Optimization* – full course; 2 credits (units)

*Graph Theory* – full course; 2 credits (units)

***Elective courses of the section M.1 “General scientific cycle. Variable part”; minimal total volume – 4 credits (units)***

*Stochastic Processes* – the first part of the course; *2* credits (units)

*Numerical Methods* – the first part of the course; *2* credits (units)

*Modern Methods of Computational Mathematics* – full course; *2* credits (units)

*Numerical Modelling of Discrete Random Processes* *and Fields* – full course; *2* credits (units)

*Methods of Discrete Simulation* – full course; *2* credits (units)

*Theory of Programming* – full course; *2* credits (units)

***Elective courses of the section M.2 “Professional cycle. Variable part”; minimal total volume – 2 credits (units)***

*Integer Programming* – full course; *2* credits (units)

*Machine Learning and Data Mining* – full course; *2* credits (units)

*Combinatorial Problems on Cayley Graphs* – full course; *2* credits (units)

*Combinatorial Designs* – full course; *2* credits (units)

***Scientific seminar (at the choice of the student) – 2 credits (units)***

***Obligatory disciplines of the section M.3 “Scientific practice and research work”; total volume – 8 credits (units)***

*Research work on the topic of master dissertation –* the first part of the discipline; 8 credits

**TOTAL MINIMUM – 24 CREDITS (UNITS)**

THE SECOND (“SPRING”) SEMESTER

***Obligatory courses of the section M.1 “General scientific cycle. Basic part”; total volume – 4 credits (units)***

*English Academic Writing* – the first part of the discipline; 2 credits (units)

*Financial Mathematics* – full course; *2* credits (units)

***Obligatory courses of the section M.2 “Professional cycle. Basic part”; total volume – 4 credits (units)***

*Operation research* – full course; *4* credits (units)

***Elective courses of the section M.1 “General scientific cycle. Variable part”; minimal total volume – 6 credits (units)***

*Stochastic Processes* – the second part of the course; *2* credits (units)

*Numerical Methods* – the second part of the course; *2* credits (units)

*Random Walks* – full course; *2* credits (units)

*Markov chains* – full course; *2* credits (units)

***Elective courses of the section M.2 “Professional cycle. Variable part”; minimal total volume – 2 credits (units)***

*Mathematical Models in Logistics* – full course; *2* credits (units)

*Approximation Algorithms* – full course; *2* credits (units)

*Routing and Scheduling*  – full course; *2* credits (units)

***Scientific seminar (at the choice of the student) – 2 credits (units)***

***Obligatory disciplines of the section M.3 “Scientific practice and research work”; total volume – 10 credits (units)***

*Research work on the topic of master dissertation –* the second part of the discipline; 8 credits

*Reports at scientific seminars and conferences –* the first part of the discipline; 2 credits (units)

**TOTAL MINIMUM – 28 CREDITS (UNITS)**

THE THIRD (“AUTUMN”) SEMESTER

***Obligatory courses of the section M.1 “General scientific cycle. Basic part”; total volume – 4 credits (units)***

*History of numerical statistical modelling and simulation* – full course; 2 credits (units)

*Number theory* – full course; *2* credits (units)

***Obligatory courses of the section M.2 “Professional cycle. Basic part”; total volume – 4 credits (units)***

*Scheduling Theory* – full course; 2 credits (units)

*Coding Theory* – full course; 2 credits (units)

*Cryptography and Cryptanalysis* – the first part of the course; 2 credits (units)

***Elective courses of the section M.1 “General scientific cycle. Variable part”; minimal total volume – 4 credits (units)***

*Modern Methods of Computational Mathematics* – full course; *2* credits (units)

*Numerical Modelling of Discrete Random Processes* *and Fields* – full course; *2* credits (units)

*Methods of Discrete Simulation* – full course; *2* credits (units)

*Theory of Programming* – full course; *2* credits (units)

***Elective courses of the section M.2 “Professional cycle. Variable part”; minimal total volume –2 credits (units)***

*Integer Programming* – full course; *2* credits (units)

*Machine Learning and Data Mining* – full course; *2* credits (units)

*Combinatorial Problems on Cayley Graphs* – full course; *2* credits (units)

*Combinatorial Designs* – full course; *2* credits (units)

***Scientific seminar (at the choice of the student) – 2 credits (units)***

***Obligatory disciplines of the section M.3 “Scientific practice and research work”; total volume – 14 credits (units)***

*Research work on the topic of master dissertation –* the third part of the discipline; 8 credits

*Development of master dissertation –* the first part of the discipline; 6 credits (units)

**TOTAL MINIMUM – 30 CREDITS (UNITS)**

THE FOURTH (“SPRING”) SEMESTER

***Obligatory courses of the section M.2 “Professional cycle. Basic part”; total volume – 4 credits (units)***

*Cryptography and Cryptanalysis* – the second part of the course; 2 credits (units)

***Elective courses of the section M.2 “Professional cycle. Variable part”; minimal total volume – 2 credits (units)***

*Mathematical Models in Logistics* – full course; *2* credits (units)

*Approximation Algorithms* – full course; *2* credits (units)

*Routing and Scheduling*  – full course; *2* credits (units)

***Scientific seminar (at the choice of the student) – 2 credits (units)***

***Obligatory disciplines of the section M.3 “Scientific practice and research work”; total volume – 18 credits (units)***

*Research work on the topic of master dissertation –* the fourth part of the discipline; 6 credits

*Reports at scientific seminars and conferences –* the second part of the discipline; 2 credits (units)

*Development of master dissertation –* the second part of the discipline; 10 credits (units)

***Obligatory disciplines of the section M.4 “Final state validation”; total volume – 12 credits (units)***

*Passing the state exam –* full discipline; 6 credits (units)

*Preparation and defending the master dissertation –* full discipline; 6 credits (units)

**TOTAL MINIMUM – 38 CREDITS (UNITS)**

**Contents of courses and disciplines of the MEP**

**«Modern Trends in Discrete Mathematics and Combinatorial Optimization»**

**Course:**

**Graph Theory**

**Author:**

**Pyatkin Artem Valeryevich**

* doctor of physical and mathematical sciences;
* leading researcher of Sobolev Institute of Mathematics SB RAS;
* Professor of the Chair of Theoretical Cybernetics in the Department of Mechanics and Mathematics of Novosibirsk State University.

**Course description**

The course «Graph Theory» presents the most important theoretical concepts of the graph theory including matching, coloring, hamiltonicity, subgraphs and minors, random graphs, etc. The course contains the proofs of the main theorems from these fields. The knowledge of these concepts is basic for further study of discrete mathematics and combinatorial optimization. Note that although almost all proofs met in the course are constructive (i.e. they can be transformed into corresponding algorithms), the algorithmic aspects of the graph theory are beyond this course.

**The aims and objectives**of the course «Graph Theory» are:

* to provide students with the basic knowledge of graphs and their properties;
* to teach students the main techniques of the proofs used in graph theory;
* to show students the applications of graphs and graph theory methods.

**Learning outcomes of the course**

As the result of study of the course «Graph Theory» the student

*should know*

* the basics of graph theory including the main definitions, graph classes, and the formulations of the most important theorems,
* proof ideas of the theorems;
* methods and techniques of graph theory including decomposition, coloring, recharging, minors and subgraphs searching, probabilistic methods;

*should be able*

* to determine the class of graphs met in applied problems;
* to apply the methods of graph theory for solving problems of discrete mathematics and combinatorial optimization;
* to use the graph models for more convenient presentation of the problems;

*should possess*the various techniques and methods for analyzing and solving graph theory problems.

**Course content**

**1. Introduction to the graph theory. Main graph classes (2 hours):** Definitions, notation and examples. Main graph classes (trees, bipartite graphs, chordal graphs, etc) and their properties.

**2. Matching theory (6 hours):** Matching in bipartite graphs. Koenig’s and Hall’s theorems. Matching in general graphs. Tutte’s theorem. Petersen’s theorem about cubic graphs. Structural Theorem of Gallai-Edmonds.

**3. Connectivity (4 hours):** Structure of 2-connected and 3-connected graphs. Menger’s theorem. Theorem of Tutte and Nash-Williams.

**4. Planar graphs (4 hours):** Embeddable graphs. Euler’s formula. Pontryagin-Kuratovski theorem.

**5. Coloring (6 hours):** History: 4-color problem. Vertex coloring. Miecelski’s construction. Brooks’ theorem. Edge coloring. Koenig’s theorem. Vizing’s theorem. List coloring. Thomassen’s theorem. Galvin’s theorem.

**6. Eulerian and Hamiltonian cycles (2 hours):** Eulerian cycles. Petersen’s 2-factor theorem. Hamiltonian cycles. Dirac’s theorem. Chvatal’s theorem.

**7. Subgraphs and minors (6 hours):** Turan’s theorem. Szemeredi’s regularity lemma. Erdős-Stone theorem. Mader’s theorem. Existence of minors in graphs of large girth and minimum degree. Hadwiger’s conjecture. Edge-maximal graphs without minors *K*4 and *K*5.

**8. Random graphs (4 hours):** Definition of random graphs.Erdős theorem. Properties of almost all graphs.

**Method of assessment**

At the end of the semester the examination is planned.

**Main literature**

#### 1. R. Diestel. Graph Theory. Springer-Verlag, Heidelberg. Graduate Texts in Mathematics. 2010.

2. L.Lowasz, M.Plummer. Matching Theory. Akadimiai Kiado, Budapest, 1986.

**Course:**

**Coding Theory**

**Author:**

**Faina Ivanovna Solov’eva**

- Full Professor, Doctor of Physical and Mathematical Sciences;

- Professor of NSU;

- Leading Researcher of IM SB RAS; see: http://math.nsc.ru/english.html

**Course description**

The course «Coding Theory (basic course)» is a basic and fundamental discipline for the section «Professional cycle» of the MEP «Modern trends in discrete mathematics and combinatorial optimization». The course is prepared for studying at the third semester of the MEP.

In the course “Coding theory” the classical and modern results from the theory of error-correcting codes in the binary and the *q*-ary symmetric channels are presented. The main topics are the following: the classical coding and decoding systems, the general theory of cyclic codes, concatenation and switching methods to construct codes, nontrivial properties of the most important classes of codes such as BCH-codes, Red-Solomon codes, Reed-Muller codes. The practical implementations of coding theory to transmit the information using communication channels with errors, implementations to cryptography are also considered in the course. To make the course self-contained the essential fundamentals concerning the presented coding theory results of the Galois field, linear algebra, combinatorics and probability are given.

***The aims and objectives*** of the course «Coding theory» are the following:

* give the basic knowledge of the theory of error-correcting codes and applications of the methods for practical implementations;
* teach students to learn the main theory of cyclic codes, the methods of coding and decoding;
* train students to construct codes having parameters close to optimal ones and good for transmission the information into communication symmetric channels with errors, understand the main practical implementations of coding theory to cryptography.

When developing the course, the long experience of the author (beginning with 1991) in teaching at Mechanical-Mathematical and Information Technology Departments of NRU NSU the disciplines concerned the theory of error-correcting codes, together with various teaching aids, are deeply used.

**Learning outcomes of the course**

As the result of study of the course «Coding Theory» a student

*should know*the essential fundamentals concerning the presented coding theory results of the Galois field, linear algebra, combinatorics and probability;

*should be able to*understand the main methods to construct codes with good parameters, especially cyclic codes;

*should possess*the algorithms of coding and decoding cyclic codes with the aim to transmit the open or hidden information into communication channels with errors.

**Course content**

1. Introduction to Galois fields.

2. Communication channel.

3. Linear codes, Hamming codes.

4. Decoding. Shannon Theorem.

5. Methods to construct codes (switchings, concatenations, other methods).

6. Perfect codes. Existence theorem. Constructions. Properties.

7. Cyclic codes. Coding, decoding.

8. BCH codes, coding, decoding.

9. MDS codes. Reed-Solomon and Justesen codes.

10. Fourier transform, MacWilliams theorem.

11. Hadamard matrices. Hadamard codes.

12. Reed-Muller codes.

13. Implementation of coding theory to transmit the information into communication channels with errors.

14. Implementation of coding theory to cryptography: APN-functions.

15. Implementation of coding theory to cryptography: MacElias cryptosystem. Niderraiter cryptosystem. Secret sharing. Autentification.

**Method of assessment**

In the program of the course we plan to provide the colloquium and the examination. Students also are offered to carry out the semester home tasks. The detailed grades for the home task, the colloquium and the examination are presented in the working program of the discipline.

**Basic literature**

1. MacWilliams F. J. and Sloane N. J. A., The Theory of Error-Correcting Codes,

North-Holland Publishing Company, 1977, 745 pp.

2. van Lint J.H., Introduction to coding theory. - 3rd rev. Springer-Verlag Berlin Heidelberg New York, 1999, 227 pp.

3. van Tilborg H., Error-correcting codes, Printed in Sweden, Studentliteratur, Lund, 1993, 235 pp.

4. Solov'eva F.I., On perfect codes and related topics, Lecture Notes, Pohang University of Science and Technology (POSTECH), Republik of Korea, 2004, 80 pp.

**Course:**

**Combinatorial Optimization**

**Author:**

**Alexander Veniaminovich Kononov**

-associated professor of the Chair of Theoretical Cybernetic in Department of Mechanics and Mathematics (DMM) of Novosibirsk State University (NSU);

-senior researcher of Laboratory of Mathematical Models of Decision making of Sobolev Institute of Mathematics of Siberian Branch (SB) of RAS; see <http://www.math.nsc.ru/LBRT/k5/lab.html>

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**Course description**

There are many reasons for studying algorithms. The primary one is to enable students to use computer efficiently. A novice programmer, without basic knowledge of algorithms, may prove to be a disaster to his company where he works. The study of algorithms is the main objective of this course.

One of the most important problems of modern society is the processing of vast amounts of data to make the best decision. One of the promises of the information technology era is that many decisions can now be made rapidly by computers. The science of how to make decisions in order to achieve some best possible goal has created the field of combinatorial or discrete optimization.

Combinatorial optimization is one of the most active areas of discrete mathematics. It has been the subject of extensive research for over fifty years. Combinatorial optimization has its roots in combinatorics, theoretical computer science and operations research. A main motivation is that hundreds or thousands of real-life problems and phenomena can be considered as combinatorial optimization problems. Therefore, the design of combinatorial algorithms now is a vast area containing many strong techniques for various applied problems.

The course «Combinatorial Optimization» includes the fundamentals of graph theory, linear and integer programming, and complexity theory. The course covers classical topics in combinatorial optimization as well as very recent results. It consists of two big parts. Most of the problems considered in the first part have efficient algorithms. Optimal solutions of these problems can be found in time polynomial in the size of the input. While most of the problems studied in the second part of the course are NP-hard. A polynomial-time algorithm for NP-hard problems is unlikely to exist. However, in many cases one can at least find approximation algorithms with provably good performance.

The course «Combinatorial Optimization» belongs to the base part of the professional cycle of MEP «Modern trends in discrete mathematics and combinatorial optimization» since it provides theoretical knowledge that is useful for studying the other courses of the MEP.

**Learning outcomes of the course**

As the result of study of the course «Combinatorial Optimization» the student

**should know**

* the typical discrete optimization problem, the combinatorial algorithms, and the formulations of the most important theorems,
* classes P and NP, the notions NP-completeness and NP-hardness;
* proof ideas of the theorems;
* methods and techniques of combinatorial optimization including greedy algorithms, dynamic programming, linear programming, matroids, flow algorithms, approximation algorithms and schemes;

**should be able**

* to determine the combinatorial complexity of new combinatorial problems;
* to apply the methods of combinatorial optimization for solving problems of discrete mathematics;
* to reduce new combinatorial problems to the problem studied in the course;

**should possess** the various techniques and methods for analyzing and solving combinatorial optimization problems.

**Course content**

1. Graphs: connectivity criteria, characterization of trees, bipartite graphs, Euler’s graphs. (4 hours)
2. Introduction in linear programming and integer linear programming. (2 hours)
3. Spanning trees and Arborescences: Kruskal’s algorithm, Prim’s algorithm, Edmonds’ branching algorithm. (2 hours)
4. Shortest Paths: Dijkstra’s algorithm, Moore-Bellman-Ford algorithm, Floyd-Warshall algorithm. (2 hours)
5. Network Flows: Ford-Fulkerson algorithm, Menger’s theorems, Edmonds-Karp algorithm, Goldberg-Tarjan algorithm. (4 hours)
6. Matching problems: Tutte matrix and Lovász randomized algorithm, Berge-Tutte theorem, Edmonds’ Cardinality Matching algorithm. (6 hours)
7. Matroids: characterization of matroids and Edmonds-Rado theorem. (2 hours)
8. Introduction in complexity theory. (classes P and NP, NP-completeness, Cook theorem) (4 hours)
9. Approximation algorithms and schemes. (2 hours)
10. Knapsack and Bin-packing problem: Weighted Median algorithm, Dynamic programming and an FPTAS, Fernandes-de-la-Vega-Lueker algorithm. (2 hours)
11. Scheduling problems: Graham’s algorithm, LPT rules, Novicki & Smutnicki algorithm. (2 hours)
12. The Traveling Salesman Problem: Double-Tree algorithm, Christofides-Serdyukov algorithm. (2 hours)

**Method of assessment**

In the program of the course, the carrying out the examination is provided.

**Basic literature**

1. B. Korte and J. Vygen. *Combinatorial optimization.* Springer, Berlin, Germany, fourth edition, 2007.

2. M.R. Garey and D.S. Johnson. *Computers and Intractability: A Guide to the theory of NP-completeness*. W.H. Freemann and Company, San Francisco, CA, 1979.

3. D.S. Hochbaum, editor. Approximation algorithms for NP-hard problems. PWS-publishing Company, Boston, MA, USA, 1997.

**Course:**

**Scheduling Theory**

**Author:**

**Alexander Veniaminovich Kononov**

-associated professor of the Chair of Theoretical Cybernetic in Department of Mechanics and Mathematics (DMM) of Novosibirsk State University (NSU);

-senior researcher of Laboratory of Mathematical Models of Decision making of Sobolev Institute of Mathematics of Siberian Branch (SB) of RAS; see <http://www.math.nsc.ru/LBRT/k5/lab.html>

- e-mail: [alvenko@math.nsc.ru](mailto:alvenko@math.nsc.ru)

**Course description**

The study of scheduling dates back to 1950s. The main topic of scheduling theory is the efficient allocation of one or more resources to activities over time. Adopting manufacturing terminology, a job consists of one or more activities, and a machine is a resource that can perform at most one activity at a time. The machines and jobs can take many different forms. The machines may be processors, runways at an airport, crews at a construction site, processing units in a computing environment, and so on. The jobs may be operations in a production process, take-offs and landings at an airport, stages in a construction project, executions of computer programs, and so on. Each job may have a certain priority level, an earliest possible starting time and a due date. The objectives can also take many different forms. One objective may be the minimization of the completion time of the last job and another may be the minimization of the number of jobs completed after their respective due dates. Scheduling theory is characterized by a huge number of problem types and is important in discrete optimization and operations research.

The course «Scheduling theory» focuses on deterministic machine scheduling. The course covers classical topics in scheduling theory as well as very recent results. It consists of three parts. The first introductory part of the course deals with the classification of scheduling problems, methods of combinatorial optimization that are relevant for the solution procedures, and computational complexity theory. The problems considered in the second part have efficient algorithms. Optimal solutions of these problems can be found in time polynomial in the size of the input. The second part covers many classical exact polynomial-time scheduling algorithms. The scheduling problems studied in the third part of the course are NP-hard. A polynomial-time algorithm for NP-hard problems is unlikely to exist. However, in many cases one can at least find approximation algorithms with provably good performance.

**Learning outcomes of the course**

As the result of study of the course «Scheduling theory» the student

**should know**

* the typical scheduling problems, the scheduling algorithms, and the formulations of the most important theorems,
* classes P and NP, the notions NP-completeness and NP-hardness;
* methods and techniques of scheduling theory including reduction of scheduling problems to the classical problems in combinatorial optimization, greedy algorithms, dynamic programming, linear programming, approximation algorithms and schemes;

**should be able**

* to determine the combinatorial complexity of new scheduling problems;
* to apply the methods of combinatorial optimization for solving scheduling problems;
* to reduce new scheduling problems to the problem studied in the course;

**should possess** the various techniques and methods for analyzing and solving scheduling problems.

**Course content**

1. Classification of scheduling problems. (2 hours)
2. Connection between scheduling problems and other problems in combinatorial optimization. (2 hours)
3. Dynamic programming. (2 hours)
4. Single machine scheduling problems: Lawler’s algorithm, EDD-rule, Smith’s rule, Moor’s algorithm. (6 hours)
5. Parallel machine scheduling problems with preemption: identical machines, uniform machines, unrelated machines. (6 hours)
6. Shop scheduling problems: open shop, flow shop, job shop. (4 hours)
7. Approximation algorithms for scheduling: parallel machines without preemptions, single machine scheduling with delivery times. (6 hours)
8. Polynomial time approximation schemes for the parallel machine scheduling problems. (4 hours)
9. Polynomial time approximation scheme for the open shop scheduling problem. (4 hours)

**Method of assessment**

In the program of the course, the carrying out the examination is provided.

**Basic literature**

1. P. Brucker. *Scheduling algorithms.* Springer-Verlag, Berlin, Germany, second, revised and enlarged edition, 1998.
2. D.S. Hochbaum, editor*. Approximation algorithms for NP-hard problems.* PWS-publishing Company, Boston, MA, USA, 1997.
3. S. Sevastianov, G. Woeginger*. Makespan minimization in open shops: a polynomial time approximation.* Mathematical programming, Series B, 82, 1998, 191-198.
4. A. Kononov, M. Sviridenko. *Linear time combinatorial approximation scheme for makespan minimization in open shop with release dates*, Operations Research Letters, 2002, v.30, p.276-280.

**Course:**

**Approximation Algorithms**

**Author:**

**Alexander Veniaminovich Kononov**

-associated professor of the Chair of Theoretical Cybernetic in Department of Mechanics and Mathematics (DMM) of Novosibirsk State University (NSU);

-senior researcher of Laboratory of Mathematical Models of Decision making of Sobolev Institute of Mathematics of Siberian Branch (SB) of RAS; see <http://www.math.nsc.ru/LBRT/k5/lab.html>

- e-mail: [alvenko@math.nsc.ru](mailto:alvenko@math.nsc.ru)

**Course description**

One of the most important problems of modern society is the processing of vast amounts of data to make the best decision. One of the promises of the information technology era is that many decisions can now be made rapidly by computers. The science of how to make decisions in order to achieve some best possible goal has created the field of combinatorial or discrete optimization.

Unfortunately, most interesting and important discrete optimization problems are NP-hard. Thus, unless P=NP, there are no efficient algorithms to find optimal solutions to such problems. If we cannot find an optimal solution efficiently then it is reasonable to settle for a near optimal solution that can be computed efficiently. Of course, we want to find solutions as close as possible to the optimum. The design of approximation algorithms is one of the most rapidly developing areas in discrete optimization problems in the last two decades. Two leading experts in the field of discrete optimization David Williamson and David Shmoys give in his book the following five reasons why you need to study the approximation algorithms.

* Because we need algorithms to get solutions to discrete optimization problems.
* Because algorithm design often focuses first on idealized models rather than the “real-world” application.
* Because it provides a mathematically rigorous basis on which to study heuristics.
* Because it gives a metric for stating how hard various discrete optimization problems are.
* Because it’s fun.

The course Approximation algorithms» introduces students to the basic ideas and approaches to the design of approximation algorithms. It consists of three big parts. In the first part we consider combinatorial algorithms for a number of important problems, using a wide variety of algorithm design techniques. In the second part we present the underlying ideas and the main tools for the construction of polynomial time approximation schemes. In the third part we cover linear programming based algorithms. We consider two fundamental techniques: rounding and the primal-dual schema.

**Learning outcomes of the course**

As the result of study of the course «Approximation algorithms» the student

**should know**

* the typical NP-hard optimization problem, the approximation algorithms for the typical NP-hard optimization problem
* classes P and NP, the notions NP-completeness and NP-hardness;
* proof ideas of the theorems;
* methods and techniques of combinatorial optimization including greedy algorithms, dynamic programming, linear programming, flow algorithms, approximation algorithms and schemes;

**should be able**

* to determine the combinatorial complexity of new combinatorial problems;
* to apply the methods of combinatorial optimization for solving NP-hard;
* to analyze the worst-case performance of algorithms;
* to construct new approximation algorithms for NP-hard optimization problems.

**should possess** the various techniques and methods for analyzing and solving combinatorial optimization problems.

**Course content**

1. Introduction. NP-hard optimization problems. Lower bounding and well-characterized problems. (2 hours)
2. Set cover. The greedy algorithm. Layering. Application to shortest superstring. (2 hours)
3. Geometric problems. The Steiner Tree problem. The Traveling Salesman problem. The k-Center problem . (4 hours)
4. Shortest superstring. (2 hours).
5. Approximation schemes. A PTAS and an FPTAS. (2 hours)
6. An asymptotic PTAS for Bin Packing. (1 hour)
7. A PTAS for Minimum Makespan Scheduling. (1 hour)
8. A PTAS for open shop problem (2 hours)
9. LP-Duality. Rounding Applied to Set Cover. Set Cover via Primal-Dual schema. The Uncapacitated Facility Location Problem. (4 hours)
10. Randomized algorithms and derandomization. Maximum Satisfiability problem. Maximum Cut problem (4 hours)
11. Semidefinite programming. Finding Large cuts. Approximating Quadratic Programs. (4 hours)
12. A local search algorithms. The Uncapacitated Facility Location Problem. The k-Median problem. (4 hours)
13. **Method of assessment**

In the program of the course, the carrying out the examination is provided.

**Main literature**

1. V. V. Vazirani. *Approximation algorithms.* Springer-Verlag, Berlin, Germany, 1998.
2. D.S. Hochbaum, editor*. Approximation algorithms for NP-hard problems.* PWS-publishing Company, Boston, MA, USA, 1997.
3. D. P. Williamson and D.B. Shmoys. *The design of approximation algorithms.* Cambridge UP, New York, NY, 2011.

**Course:**

**Operations Research**

**Authors:**

**Adil Ilyasovich Erzin, Ivan Ivanovich Takhonov**

Adil I. Erzin **–** Doctor of Sciences; Full Professor; Head of Chair of Theoretical Cybernetics of NSU; Leading Researcher of IM SB RAS;

Ivan Takhonov – Candidate of Sciences (PhD), Assoc. Prof. of Chair of Theoretical Cybernetics of NSU.

**Course description**

This course is designed to introduce students to basic models, concepts, and methods of Operations Research (OR). The course starts with brief introduction to computational complexity and description of some basic OR models. In the following discussion, some common methods to deal with optimization problems are observed (such as dynamic programming, enumeration methods) and several important graph optimization problems are considered (network flow problems, matching and assignment problems). Some “non-optimization” models (project planning and analysis, game-theoretic models) are considered as well. The course concludes with brief observation of some approximation and heuristics techniques. “Operations Research” course provides basic knowledge for more advanced courses of the Master’s Program.

**Learning outcomes of the course**

At the end of the study the students *should know*:

* the place of Operations Research among other branches of discrete and continuous mathematics dealing with decision management and optimization;
* a wide range of Operations Research models and problem formulations applicable in different fields of human activity;
* algorithmic and computational aspects of OR models;
* common methods and approaches to solve OR optimization and non-optimization problems.

At the end of the study the students *should be able to*:

* use the terminology common to the field of discrete mathematics, operations research and decision making;
* give full and correct proofs of mathematical statements;
* accurately translate real-world problems into adequate OR models;
* perform the complexity analysis of a problem, prove its NP-completeness/hardness;
* use different techniques to find exact and approximate solutions of optimization problems;
* investigate structural properties of optimization problems and devise new algorithms to solve them;
* analyse performance aspects of an algorithm (such as its time and space complexity, approximability, etc.), identify performance bottlenecks and give performance improvement recommendations.

At the end of the study the students *should possess***:**

* skills in formalizing and modeling a variety of decision-making processes;
* skills in investigating discrete and continuous optimization problems;
* skills in algorithm development and analysis.

**Course Content**

The lecture part of the course includes 17 lectures (34 hours overall) divided into 9 sections and 17 practical classes (34 hours overall) to students practice in applying algorithms and method described in the lectures. The detailed description is given below. Some particular themes can be replaced or modified by including some new results.

Lecture Course (34 hours)

*Section 1. Introduction (2 hours)*

1.1. Mathematical Models in OR.

*Section 2. Computational (time) Complexity (4 hours)*

2.1. Introduction to Computational Complexity Theory.

2.2. Using NP-completeness to analyze problems.

*Section 3. Dynamic Programming (4 hours)*

3.1. Bellman’s Optimality Principle. Dynamic Programming.

3.2. Knapsack Problem. The Nearest Neighbor Problem.

*Section 4. Project Planning (4 hours)*

4.1. Project Network models and their characteristics.

4.2. Computation of the project’s parameters.

*Section 5. Implicit Enumeration (4 hours)*

5.1. Branch and Bound method.

5.2. Balas additive algorithm.

*Section 6. Matchings and Assignments (4 hours)*

6.1. Matching and Vertex Cover. Maximum Matching in a bipartite graph.

6.2. Assignment Problem.

*Section 7. Basics of Game Theory (4 hours)*

7.1. Matrix Game. Pure Strategy and Nash Equilibrium.

7.2. Mixed Strategies.

*Section 8. Network Flow Problems (4 hours)*

8.1. Maximum Flow Problem.

8.2. Minimum Cost Flow Problem..

*Section 9. Approximation Algorithms (4 hours)*

9.1. Approximation Algorithms and their characteristics.

9.2. Heuristics and Metaheuristics.

Practical Course (34 hours)

1. Mathematical modeling in OR.
2. Computational complexity. Algorithm performance analysis. Polynomial reducibility and NP-completeness.
3. Proving NP-completeness and inapproximability results.
4. Dynamic Programming. The Shortest Path Problem. The Nearest Neighbor Problem.
5. Distribution of Effort Problem. Boolean Knapsack Problem (BKP). Inverse BKP.
6. Project Planning. Project network models. Network simplification.
7. The project’s critical parameters. Bellman-Ford algorithm.
8. Branch and Bound for the Traveling Salesman Problem.
9. Balas additive algorithm for the Integer Linear Programming.
10. Maximum Matching Problem. The labeling algorithm.
11. The Assignment Problem. The Hungarian algorithm.
12. Matrix Games: the basic concepts. Pure-strategy Nash Equilibrium
13. Mixed Extension and Mixed-strategy Equilibrium. Methods of solving games.
14. Network Flows. Max-Flow Problem. Ford-Fulkerson algorithm.
15. Min-Cost Flow Problem. Klein’s and Busacker-Gowen algorithms.
16. Approximation algorithms. Performance analysis.
17. Approximation schemes.

**Method of assessment**

In the course, the following types of formal assessment are used: questioning and homework (weekly), and computational assignment (semester task). Final assessment is performed at the end of the semester through a written test and oral examination.

To grade students, a point rating system is used. In the semester a student may earn:

* up to 20 credit points for in-class work;
* up to 10 points for the computational assignment;
* up to 10 points for the written test;
* some additional points for conference or seminar participation.

The points are summed up and preliminary grades are offered. Preliminary grading criteria:

* Excellent (A) – total score at least 25 points, at least 6 points for the test, and 10 points for the computational assignment;
* Good (B) – total score at least 20 and 10 for the computational assignment;
* Satisfactory (С) – total score at least 15, or total score ≥ 11 and ≥ 1 for in-class work;
* Unsatisfactory (F) – otherwise.

If a student disagrees with his/her preliminary grade, he/she goes to exam. Exam grading criteria:

* Excellent (A) – student gives full and correct answers to the examiner’s questions, solves additional tasks, uses terminology correctly, demonstrates detailed knowledge of the course’s structure and logic;
* Good (B) – student gives correct answers to most of the questions, faces some difficulties in solving additional tasks, uses terminology correctly, demonstrates good knowledge of the course’s structure and logic;
* Satisfactory (С) – student fails to answer some important questions or cannot solve additional task, understands the course’s structure and logic within a specific domain but finds it difficult to point out the connection between different course’s parts, the usage of terminology is mostly correct;
* Unsatisfactory (F) – student fails to answer questions concerning basic terms of the course.

**Basic Literature**

1. Wolsey L.A. *Integer Programming*, John Wiley & Sons, New York (1998).
2. Garey M., Johnson D.S. *Computers and Intractability: A Guide to the Theory of NP-completeness*, W.H. Freeman and Co., San Francisco (1979).
3. Hu T.C. *Integer Programming and Network Flows*, Addison-Wesley, (1969).
4. Ford L.R., Fulkerson D.R. *Flows in Networks*, Princeton University Press (1962)

**Course:**

**Cryptography and Сryptanalysis**

**Author:**

**Natalia Nikolaevna Tokareva**

* Senior lecturer at the department of theoretical cybernetics of Novosibirsk State University.
* Researcher (senior staff scientist) at the Laboratory of Discrete Analysis in the Sobolev Institute of Mathematics (Siberian Branch of the Russian Academy of Sciences).
* PhD (2008) in mathematics (speciality: discrete mathematics and mathematical cybernetics).

**Course description**

The course «Cryptography and cryptanalysis» is a fundamental discipline for all section «Professional cycle» of the MEP «Modern trends in discrete mathematics and combinatorial optimization» and intends for studying at the fourth semester of the MEP. This course covers the foundations of the modern cryptography and cryptanalysis. It is supposed that students are familiar with main results of information theory. In this course we study

* foundations of information theory (processing of continuous information; discretization methods; information measurement; message complexity; data sources);
* problems of information storage (reliability and fail-safety, RAID-arrays; security problems);
* cryptography (Shannon secrecy theory; mathematical foundations of cipher constructing; block and stream ciphers; Boolean functions in cryptography; asymmetric cryptographic systems; cryptographic protocols);
* cryptanalysis (theoretical and practical results; statistical and analytic methods of cryptanalysis; cryptanalysis of asymmetric systems; etc);
* pseudorandom sequences and statistical methods of their analysis;
* practical aspects: secure data transmission in mobile systems and network protocols.

The course contains theoretical part (lectures) and the practical one (seminars, laboratory works). We discuss also several open problems in cryptography (especially for students that are interested in specialization in cryptography).

**Learning outcomes of the course**

At the end of the course «Cryptography and cryptanalysis» the student

*should know*the basics of information theory, cryptography and cryptanalysis;

*should be able to*analyze the methods of ciphering in terms of their resistance to basic methods of cryptanalysis; to analyze the properties of Boolean functions used in the ciphers; to choose acceptable parameters for applications of the widespread cryptographic schemes;

*should possess*the modern technologies of cryptography and cryptanalysis.

**Course content**

Theoretical part of the course includes the following items to study:

1. Foundations of information theory: processing of continuous information; discretization methods; information measurement; message complexity; data sources;
2. Problems of information storage: reliability and fail-safety, RAID-arrays; security problems;
3. Introduction into cryptography mathematical foundations of cipher constructing; block and stream ciphers;
4. Shannon secrecy theory;
5. Boolean functions in cryptography: algebraic and correlation immune functions; bent functions; AB, APN-functions;
6. Asymmetric cryptographic systems; cryptographic protocols;
7. Cryptanalysis: theoretical and practical results;
8. Statistical methods of cryptanalysis: linear and differential cryptanalysis;
9. Algebraic cryptanalysis;
10. Cryptanalysis of asymmetric systems;
11. Pseudorandom sequences and statistical methods of their analysis;
12. Practical aspects: secure data transmission in mobile systems and network protocols.

**Method of assessment**

During semester students should complete tasks on cryptography and cryptanalysis and collect scores for them. At the end of the course the examination is provided. The detailed grades for the home tasks and the examination are presented in the program of the course.

**Basic literature**

1. Tokareva N.N., *Symmetric cryptography. A short course*, Novosibirsk State University, Novosibirsk, 2012, ISBN: 978-5-4437-0067-0, 234 p.
2. Tokareva N.N., *Nonlinear Boolean functions: bent functions and their generalizations*, LAP LAMBERT Academic Publishing, Saarbrucken, Germany, 2011, ISBN: 978-3-8433-0904-2, 180 p.
3. Agibalov G.P. *Selected theorems of the initial cryptography course* // tutorial. Tomsk State University, 2005.
4. Alferov A.P., Zubov A.Yu., Kuzmin A.S., Cheremushkin A.V. *Foundations of cryptography* // M.: Geleous ARV, 2005. 480 p.
5. Logachev O.A., Salnikov A.A., Yashenko V.V. *Boolean functions in coding theory and cryptology* // M.: MCNMO, 2004. 470 p. ISBN 5-94057-117-4.
6. Moon T. K., *Error Correction Coding, Mathematical Methods and Algorithms*. Wiley, ISBN 0-471-64800-0. 2005.
7. Fomichev V.M. *Discrete mathematics and cryptology. Course of lectures* // M.: Dialog-MIFI, 2010. ISBN 5-86404-185-8.

**Course:**

**Integer Programming**

**Authors:**

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Adil I. Erzin **–** Doctor of Sciences; Full Professor; Head of Chair of Theoretical Cybernetics of NSU; Leading Researcher of IM SB RAS;

Alexander V. Plyasunov **–** Candidate of Sciences; Associate Professor of Chair of Theoretical Cybernetics of NSU; Senior Researcher of IM SB RAS;

**Course description**

Students will master the mathematics of discrete optimization which has the broad applicability in physics, statistics, transportation and distribution logistics, production scheduling, telecommunication, information, and other areas of industry. They will be familiar with state-of-the-art exact and approximate algorithms to solve optimization problems. This course gives the basic rules which should be applied to convert optimization problem into mathematical program. The focus is on understanding the mathematical underpinnings of the algorithms that make it possible to solve (exactly or approximately) the large and complex models that arise in practical applications.

To attend this course, students should have a basic understanding of algorithms and programming skills, and be familiar with fundamental optimization methods.

**Learning outcomes of the course**

As the result of study of the course «Integer Programming» the student

*should have an idea* about the place and role studied discipline among other Sciences;

*should know* the content of the course, the terms of reference, methods of investigation;

*should be able* to apply in practice, the specific computational methods to the analysis and solution of optimization problems.

**Course content**

Syllabus

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| --- |
| Section 1.  1.1. Integer programming problem  1.2. Examples of combinatorial optimization problems  1.3. Choices in modal formulations  Section 2. Computational complexity  2.1. Definitions  2.2. Measuring algorithm efficiency and problem complexity  2.3. Classes P, NP and PO, NPO  2.4. Complexity and Polyhedra  Section 3. Linear equations and inequalities  3.1. Linear algebra and computational complexity  3.2. Main theorem on linear inequalities.  3.3. Lemma Farkash  Section 4. Theory of polyhedral sets  4.1. Definition of polyhedral and dimension  4.2. Describing polyhedra by facets  4.3. Describing polyhedra by extreme points and extreme rays  Section 5. The theory of valid Inequalities  5.1. Generating all valid Inequalities  5.2. Gomory’s Fractional Cuts and Rounding  5.3. Superadditive functions and valid Inequalities  5.4. Valid Inequalities for mixed-integer sets  5.5. Superadditivity for mixed-integer sets  Section 6. Strong valid Inequalities and facets for Structured Integer Programs  6.1. Valid Inequalities for the 0-1 Knapsack Problem  6.2. Valid Inequalities for the simmetric Traveling Salesman Problem  6.3. Valid Inequalities for Variable Upper-Bound Flow Models  Section 7. Duality and Relaxation  7.1. . Duality and Value Function  7.2. Superadditive Duality  7.3. Lagrangian relaxation and Duality  7.4. Benders’ Reformulation  Section 8. General Algorithms  8.1. General Cutting-Plane Algorithms  8.2. Methods of Branch-and-Bound  8.3. Methods of Branch-and-Cut  Section. 9. Special-Purpose Algorithms  9.1. A Cutting-Plane Algorithm Using Strong valid Inequalities  9.2. Primal and dual Heuristic Algorithms  9.3. Decomposition Algorithms  9.4. Dynamic Programming |

**Method of assessment**

During the course, on each lection a number of short questions is provided. In the end of course an extensive examination is supposed.

**Basic literature**

1. G.L. Nemhauser, L.A. Wolsey. Integer and Combinatorial Optimization. John Wiley \& Sons, New York, 2011.
2. A. Schrijver. Combinatorial Optimization: Polyhedra and Efficiency. Springer, Berlin, Vol. A, B, C, 2003.
3. U. Faigle, W. Kern, G. Still.Algorithmic Principles of Mathematical Programming. Springer, Netherlands, 2002.

**Course:**

**Machine Learning and Data Mining**

**Author:**

**Vladimir Borisovich Berikov**

* Leading Researcher in Sobolev Institute of mathematics, Siberian Branch of Russian Academy of Sciences;
* Doctor of Technical Sciences;
* Associate Professor of Chair of Theoretical Cybernetics of NRU NSU.

**Course description**

The purpose of the course «Machine learning and data mining» is to provide students with basic theoretical knowledge and practical skills necessary for successful professional activity in various fields related to data analysis. The content of the course includes the main directions in modern data analysis theory: pattern recognition, prediction of quantitative variables, cluster analysis, feature selection.

*The aims and objectives* of the course “Machine learning and data mining” are:

* to give to students basic knowledge in the theory, methods and applications of intellectual data analysis;
* to teach students to apply the studied methods and algorithms for the solution of applied data analysis problems;
* to train students to realize data analysis algorithms on computer.

**Learning outcomes of the course**

As the results of study of the course “Machine learning and data mining” the students

**should know**

* main principles of the construction of mathematical models for data analysis and making optimal decisions on their basis (methods of model specification, parameter estimation and validation of results),
* methods and techniques of data analysis and the range of problems in which these methods are applicable;

**should be able**

* to apply the methods of machine learning and data mining for solving problems related to data analysis;

**should possess** the various techniques and methods for data analysis with use of specialized mathematical methods and computer packages.

**Course content**

1.*Introduction to data analysis:* Main definitions, notations, types of data analysis problems. Main directions in machine learning and data mining.

2.*Methods of discriminate analysis:* Fundamentals of pattern recognition theory. Discriminant (decision) function. Risk, probability of misclassification. Optimal (Bayesian) decision function. Optimal classifier in case of multidimensional normal distributions (two classes, equal or unequal covariance matrices). Naive Bayes classifier. Construction of classifier for binary and categorical variables. Bahadur expansion. Estimation of distribution mixture. EM algorithm. Nonparametric evaluation of density function. Parzen’s window. Nearest neighbors method. Linear classifiers. Fisher linear discriminant.

3.*Advanced pattern recognition methods:* Support vector machines. Introduction to neural networks in pattern recognition. Methods of pattern recognition based on logical rules. Logical decision functions. Decision trees. Algorithms ID3, C4.5, CART. Recursive algorithm of decision tree construction. Classifier ensemble construction. Algorithm AdaBoost. Boosting of decision trees. Random forests and its characteristics.

4.*Classifier performance evaluation:* Problem statement. Overfitting effects. Estimation of classifier accuracy with use of normal distribution model. Vapnik-Chervonenkis bounds. Bayesian estimates of classification accuracy in discrete case. Bootstrap estimations. ROC curves analysis.

5. *Quantitative variables prediction:* Introduction to regression analysis. Basic regression models. Estimation of regression model parameters. Problems of multicollinearity, heteroscedasticity and autocorrelation. Regression with categorical variables. Logistic regression model. Regression trees. Analysis of quantitative characteristics changing in time. Trend and seasonal models. Autoregression and moving average models. Methods of multidimensional heterogeneous time series analysis.

6. *Cluster analysis:* Introduction to cluster analysis. Distances between objects and between clusters. Main approaches to clustering. Clustering algorithms: *k-*means algorithm, “Forel” algorithm, hierarchical algorithm of dendrogram construction. Taxonomic decision trees. Clustering ensembles.

7. *Feature selection methods:* Measures of feature subsets quality. Approximate algorithms of searching for the best feature subset. Algorithm of adaptive random search for the most informative features.

**Method of assessment**

The program of the course provides for the examination. The detailed grade for this examination is presented in the working program of the discipline.

**Basic literature**

1. Bishop C.M. Pattern Recognition and Machine Learning. Springer, 2006.
2. Duda, R.O., Hart, P.E., Stork, D.G. Pattern classification, 2nd edn. Wiley, 2001.
3. Box G., Jenkins G. Time Series Analysis: Forecasting and Control. San Francisco: Holden-Day. 1970.
4. Jain A.K., Dubes R.C. Algorithms for Clustering Data. Prentice Hall, 1988.

**Course:**

**Combinatorial Problems on Cayley Graphs**

**Author:**

**Elena Valentinovna Konstantinova**

- Senior Researcher, PhD in technical sciences;

- Assistant Professor of the Chair of Theoretical Cybernetics at Novosibirsk state University;

(see <http://www.nsu.ru/exp/en/education/mechanics_and_mathematics/theoretical_cybernetics>);

- senior researcher at [Sobolev Institute of Mathematics](http://math.nsc.ru).

**Course description**

The course «Combinatorial Problems on Cayley Graphs» is a variable course of the MEP «Modern Trends in Discrete Mathematics and Computational Optimization» which is intended for studying at the third semester of the MEP. For studying the course some basic knowledge in the fields of graph theory, group theory, algebraic methods, combinatorics, and programming is necessary.

The course “Combinatorial Problems on Cayley Graphs” consists of selected combinatorial problems on Cayley graphs. It presents detailed descriptions of the main theoretic and algorithmic approaches in investigations of Cayley graphs. It shows connections of Cayley graphs with applied problem arising in computer sciences and bioinformatics. In the frame of this course the fundamentals of graph theory, group theory, algebraic graph theory, combinatorics are also given. The introductory lecture of the course gives the historical perspective on the development of Cayley graphs. The final lecture of the course gives the list of open problems in the theory of Cayley graphs.

**The aims and objectives**of the course «Combinatorial Problems on Cayley Graphs» are:

* to give to students the basic knowledge of the theory of Cayley graphs;
* to show students main applications of Cayley graphs;
* to show students the interdisciplinary matter in solving some problem on Cayley graphs;
* to train students in theoretical approaches to prove group-theoretical and graph-theoretical results on Cayley graph;
* to train students in algorithmic approaches to get theoretical results on Cayley graph (Gates-Papadimitrou algorithm and its modifications);

In developing the course, the vast experience of the author in teaching together with various teaching aids (Lecture Notes published by author in English) is used. Namely, the course is based on the lectures that the author delivered at Department of Mechanics and Mathematics in Novosibirsk State University, Russia; at Faculty of Mathematics, Natural Sciences and Information Technologies in University of Primorska, Slovenia; at College of Science in Yeungnam University, South Korea; at Institute for Research in Fundamental Sciences, Iran.

**Learning outcomes of the course**

As the result of study of the course «Combinatorial Problems on Cayley Graphs» the student:

* *should know*the fundamentals of theory and applications of Cayley graphs;
* *should be able to*use theoretic approaches to prove group-theoretical and graph-theoretical results on Cayley graph;
* *should be able to*use algorithmic approaches to get theoretical results on Cayley graph;
* *should be able to*understand the interdisciplinary matter of Cayley graphs;
* *should possess*techniques and methods for analyzing and solving problems on Cayley graphs.

**Course content**

1. Groups and graphs. Symmetry and regularity of graphs: vertex-transitivity, edge-transitivity, distance-regularity, distance-transitivity. (4 hours)
2. Some families of Cayley graphs (the multidimensional torus, the hypercube, the butterfly graph, the Hamming graph, the Kneser graph). Cayley graphs on the symmetric group. Networks and Cayley graphs. (4 hours)
3. Hamiltonicity of Cayley graphs. Hypercube graphs and a Gray code. Combinatorial conditions for Hamiltonicity. (2 hours)
4. Lovász and Babai conjectures. Mohar conjecture. The main results for abelian, dihedral groups of special orders. (2 hours)
5. Hamiltonicity of Cayley graphs on finite groups: a proof by Pak and Radoicic. (2 hours)
6. Hamiltonicity of Cayley graphs on the symmetric group. Hamiltonicity of the Pancake graph: two proofs. (2 hours)
7. Hierarchical and cycle structures of the Pancake graph. (4 hours)
8. The diameter problem: abelian and non-abelian groups. (2 hours)
9. Pancake problem. Algorithm by Gates and Papadimitrou. Upper and lower bounds on the diameter of the Pancake graph. Improved bounds by Heydari and Sudborough. Exact values on the diameter of the Pancake graph. (8 hours)
10. Burnt Pancake problem. Bounds and exact values for the diameter of the Burnt Pancake graph. (2 hours)
11. Sorting by reversals: signed and unsigned cases. Algorithmic approaches. Approximation algorithms. (2 hours)

**Method of assessment**

According to the program of the course, the examination is planned. Students also are offered to do some home tasks. The detailed grades for home tasks and an examination are presented in the working program of the discipline.

**Basic literature**

1. L. Babai, Automorphism groups, isomorphism, reconstruction, Handbook of combinatorics, Vol. 2, MIT Press, Cambridge, MA, 1996, 1447-1540.

2. L.W. Beineke, R. J. Wilson, and P. J. Cameron, Topics in algebraic graph theory, Series: Encyclopedia of mathematics and its applications, Vol.102, 2004.

3. A. E. Brouwer, A. M. Cohen, and A. Neumaier, Distance-regular graphs, Springer-Verlag, Berlin, Heidelberg, 1989.

4. L. Heydemann, Cayley graphs as interconnection networks, in: Graph symmetry: algebraic methods and applications, (G. Hahn, G. Sabidussi, eds.), Kluwer, Amsterdam, 1997.

5. E. Konstantinova, Lecture notes on some problems on Cayley graphs, Koper: Knjiznica za tahniko, medicino in naravoslovje, TeMeNa, 2012, 93 pp. (ISBN 978-961-93076-1-8) (for an electronic version see: <http://temena.famnit.upr.si/files/files/Lecture_Notes_2012.pdf>)

6. P. A. Pevzner, Computational molecular biology: an algorithmic approach, The MIT Press, Cambridge, MA, 2000.

**Course:**

**Mathematical Models in Logistics**

**Authors:**

**Ekaterina Vyacheslavovna Alekseeva, Yuri Andreevich Kochetov**

**Ekaterina Alekseeva**

- Associate Professor at NRU NSU;

- PhD in Physical and Mathematical Sciences;

- Researcher at IM SB RAS;

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**Yury Kochetov**

- Professor at the NRU NSU;

- Doctor of Physical and Mathematical Sciences;

- Leading Researcher at IM SB RAS;

Homepage: <http://www.math.nsc.ru/LBRT/k5/kochetov.html>

**Course description**

The course «Mathematical Models in Logistics» is devoted to the fundamental discrete combinatorial problems which have the applicability in transportation and distribution logistics, production scheduling, telecommunication, information, and other areas of industry. It presents the mathematical programming models of basic problems, such as the traveling salesman, bin-packing, vehicle routing, facility location, lot sizing and inventory problems. The course provides state-of-the-art exact and approximate algorithms, and some optimization packages to solve these problems.

Every time when we use a GPS navigator, mobile phone, or shop at our neighborhood mall, read our emails or fly for business or for vacation, we use a system that has routed paths, messages, goods or people from one place to another. That is the logistic processes are all around us. Every day we take solutions of logistics problems and the question arisen is how to find them? The underlying problems are conceptually simple, yet mathematically complex and challenging. How can we best route goods or people from one place to another? Or, how can we locate in the best way facilities to provide services and goods as efficiently as possible? The problems encountered in answering these questions often have an underlying combinatorial structure. It means that problems could be theoretically solved by enumeration approaches but models for these problems often are very large with hundreds or thousands of constraints and variables. So, direct approaches are not applicable starting with the small size of models. Over the past three decades, researchers from operations research community have made enormous progress in developing theory, models and solution methods for these problems. This course describes the exciting challenges of the field of mathematical models in logistics and shows the results of the progress in modeling and solving these problems. Furthermore students will be explained the aspect of modeling and solving real-world logistics problems using advanced modeling systems and software. The efficiency of algorithm used depends on the several aspects such as the size of a problem, the structure of data, mathematical model, algorithm implementation and others. By the end of the course students have the feeling of these aspects by considering logistics problems and the knowledge of how to manage with them. This knowledge provides a bridge between practitioners and researchers working on supply chain management.

# The main goals of the course «Mathematical Models in Logistics» are the following:

* to learn students mathematical modeling fundamental optimization problems arising in logistics;
* to learn students how to solve these problems by known algorithms;
* to learn students how to analyze algorithms in terms of their computational efficiency and possibility of their use in applied problems.

**Learning outcomes of the course**

The results of studying the course «Mathematical Models in Logistics» are the following:

student **should know**

* the basic formulation of the logistics problems and their mathematical models in terms of mathematical programming;
* the main definitions of the most important concepts (feasibility, optimality, computational complexity of problem and algorithm, mathematical model classification, taxonomy of the algorithms and problems considered);
* the proof ideas of the theorems;
* methods and techniques applied to solve the problems considered in the course;

student **should be able**

* write down mathematical programming models of logistics problems considered in the course;
* analyze the mathematical models with respect to type of constrains, variables;
* to determine the class of algorithms applied to solve these problems;
* to apply the methods of combinatorial optimization, mathematical programming for solving logistics problems;
* to implement algorithms, make numerical experiments, and analyze its results;
* to use the modern modeling languages and systems to solve optimization problems;

student **should possess** the various techniques and methods for analyzing and solving logistics problems.

**Course content**

1. Introduction to logistics problems. Production planning by mixed integer programming. Facility location problems: p-median problem, p-median problem with clients’ preferences.

2. Mixed integer programming models and their analysis. Multistage uncapacitated facility location problem. Lower bounds on the optimal value. Probabilistic greedy algorithms.

3. Modeling system languages and modeling software.

4. Dynamic logistics management. Primal and inverse location problems over a fixed finite planning horizon. Lagrangian relaxation. Dynamic capacitated location problems.

5. Competitive production planning models. Concept of bi-level optimization. Stackelberg games, leader-follower problem. Upper bounds on the optimal value and exact method. Competitive location problems.

6. Transportation logistics models. Traveling salesman problem and related problems. Vehicle routing problems. Metaheuristics approaches.

7. Warehouses logistics models. Bin-packing problems. Approximate, asymptotic and exact algorithms. 2D rectangle and circle packing problems. Column generation approach for 2D packing problems.

8. Optimization with multiple criteria. Efficient solutions and non-dominated points. Weighted sum method. Epsilon constrained method. Two phase method. Metaheuristics approaches. Leader-follower problem with multiple criteria.

9. Multi-product lot-sizing and scheduling problem. Mixed integer linear program. Genetic algorithm.

**Method of assessment**

In the program of the course we plan to provide the oral examination. Students also are offered to carry out the semester home tasks. The detailed grades for the home task, and the examination are presented in the working program of the discipline.

**Basic literature**

1. Talbi El-Ghazali Metaheuristics: From Design to Implementation. John Wiley & Sons, 2009
2. Williamsom D., Shmoys D. The Design of Approximation Algorithms. Cambridge University Press, 2011.
3. Cormen T.H., Leiserson C.E., Rivest R.L., Stein C. Introduction to Algorithms. MIT Press, Cambridge, MA, USA, fourth edition, 2013.
4. Chvatal V. Combinatorial Optimization. Methods and Applications. IOS Press, 2011.
5. Golden B., Raghavan S., Wasil E. The Vehicle Routing Problem: Latest Advances and New Challenges. Springer, 2010.

**Internet resources:**

1. Discrete Location Problems Benchmark Library

<http://math.nsc.ru/AP/benchmarks/english.html>

1. The General Algebraic Modeling System <http://gams.com/>
2. European Working Group on Location Analysis <http://www.euro-online.org/ewgla/>
3. European Working Group on Vehicle Routing and Logistics Optimization <http://www.verolog.eu/>
4. The Metaheuristics Community <http://www.metaheuristics.eu/>
5. The Institute for Operations Research and the Management Sciences (INFORMS) is the largest professional society in the world for professionals in the field of operations research, management science, and analytics <https://www.informs.org/>
6. Association of European Operational Research Societies <http://www.euro-online.org/web/pages/1/home>
7. The Travelling Salesman Problem <http://www.math.uwaterloo.ca/tsp/>

**Course:**

**Combinatorial Designs**

**Author:**

**Ivan Yuryevich Mogilnykh**

- Ph.D in Mathematics in 2010;

- Assistant of professor of Novosibirsk State University;

- Research fellow of IM SB RAS;

**Course description**

The course «Combinatorial Designs» is a fundamental discipline for the section «Professional cycle» of the MEP «Modern trends in discrete mathematics and combinatorial optimization». The course is prepared for studying at the third semester of the MEP.

The basic theoretical aspects of design theory are outlined in the course “Combinatorial designs”.

The following topics are considered: necessary existence conditions, Fisher inequality, construction of Steiner triple and quadruple systems, symmetric designs, the difference set method, an overview of the designs constructed from graphs, codes, projective geometries etc. The information about the practical implementation of the presented theory along with the PC realization aspects are also presented. In order to make the course self-contained a necessary mathematical background on combinatorics and graphs is given.

***The aims and objectives*** of the course «Combinatorial designs» are the following:

* to give the basics of the theory of combinatorial designs and its applications for practical implementations;

- the development the mathematical combinatorial skills of students, being advanced and beginners in the topic. Generally speaking, popularization of discrete mathematics and combinatorics.

* to form the tight interrelation of the theoretical facts and the practical implementation of combinatorial objects (designs) in the mindset of the students.

**Learning outcomes of the course**

As the result of study of the course «Coding Theory» a student

*should know*the essential fundamentals concerning the aspects of the graph theory and combinatorics presented in the course;

*should be able to*understand the main methods of constructing designs, especially Steiner triple and quadruple systems;

*should possess*the various techniques of proving nonexistence of combinatorial designs, including computer-based ones.

**Course content**

1. Introduction to design theory.

2. Necessary conditions for existence of t-designs: integer conditions, Fisher inequality and Mann theorem.

3. Symmetric designs. Brook-Ryser-Chowla theorem. Symmetric Steiner systems and projective planes. Dual designs and their parameters.

4. Constructions of Steiner Triple Systems: Assmuss-Mattson constructions, direct product construction, Moore construction. Bose and Skoolem constructions, commutative idempotent and half-idempotent latin squares. The sufficiency of integer conditions for Steiner triple systems.

5. Steiner triple systems and perfect codes. Lower and upper bounds on Steiner triple systems.

6. Constructions of quadruple Steiner systems. Hannani and Aliev constructions. Switchings of Steiner triple and quadruple.

7. Alltop constructions.

8. The difference set construction.

9. Designs from graphs, groups, Hadamard matrices, etc.

10. Designs and error-correcting codes: Preparata, Golay, Hadamard, BCH and other codes. Designs and completely regular codes in Hamming and Johnson schemes.

11. Witt designs and Mattieu groups.

12. Designs in practice: from designs of experiments, problems of optimization to cryptography and hardware.

13. Combinatorial objects with similar properties: orthogonal arrays, q-ary Steiner systems.

14. Realization of design theory problems on PC. Computer algebras: GAP, Magma etc.

**Method of assessment**

In the program of the course we plan to provide the colloquium and the examination. Students also are offered to carry out the semester home tasks. The detailed grades for the home task, the colloquium and the examination are presented in the working program of the discipline.

**Main literature**

1. T. Beth, D. Jungnickel, H. Lenz. Design theory, second edition, Cambridge university press, UK, 1999.

2. M. Hall. Combinatorial theory, Wiley and sons, Canada, 1998.

3. C.J. Colbourn, J.H. Dinitz, CRC Handbook of Combinatorial Designs, CRC Press, New York, 1996.

4. F.J.MacWilliams, N.J.A. Sloane, The Theory of Error-Correcting Codes, North-Holland Mathematical Library, New York, 1977.

**Course:**

**Routing and Scheduling**

**Author:**

**Ilya Dmitryevich Chernykh**

* candidate of physical and mathematical sciences
* associate professor of the Chair of Theoretical Cybernetics in Department of Mechanics and Mathematics (DMM) of Novosibirsk State Univercity (NSU)
* senior researcher of Laboratory of Discrete Optimization in Operations Research of Sobolev Institute of Mathematics of Siberian Branch of RAS

**Course description**

The course «Routing and Scheduling» concerns the combination of shop scheduling problems with classic metric TSP. It contains basic information on classic shop scheduling models (open shop and flow shop) as well as recent results on the routing shop scheduling problems.

The aim of the course is to introduce modern techniques and ideas for researching of shop scheduling problems and designing of efficient algorithms on the example of routing scheduling problems.

The corresponding objectives of the course «Routing and Scheduling» are:

* to give students knowledge on algorithmic complexity and approximability of classic shop scheduling problems and their respective routing versions;
* to teach students to develop approximation algorithms for scheduling problems and perform their worst-case analysis;

The course is based on the author's joint recent scientific articles and his experience in teaching the disciplines related to scheduling at the Department of Mechanics and Mathematics of NRU NSU.

Being a natural extension of the courses «Scheduling theory» and «Approximation Algorithms», the course «Routing and Scheduling» can be used as a part of international master programme «Modern Trends in Discrete Mathematics and Combinatorial Optimization»/

**Learning outcomes of the course**

As a result of study of the course «Routing and Scheduling» the student

*should know* basic knowledge on properties of classic and routing shop scheduling problems, their algorithmic complexity and best known up to date approximation algorithms;

*should be able to* analyze approximation algorithms for shop scheduling problems and investigate optima localization intervals for those problems in terms of trivial lower bounds;

*should possess* the methods and approaches for developing efficient algorithms for shop scheduling problems.

**Course content**

Course content

1. Traveling salesman problem: complexity, approximation and basic polynomially solvable cases. Christofides-Serdyukov algorithm for the metric TSP.
2. Shop scheduling models and feasible schedules. Standard lower bound for the optimum. Optima localization problem.
3. Open shop: complexity, approximation and polynomially solvable cases. Normal schedules and normal instances. Gonzalez-Sahni algorithm for the case of two machines. Dense schedules and their properties. Greedy algorithm and Chen-Strusevich conjecture. Optima localization for the case of three machines.Flow shop: complexity, approximation and polynomially solvable cases. Johnson's algorithm. Averbakh-Berman approximation heuristic.
4. Branch-and-bound method and computer-aided research for scheduling models.
5. Complexity and approximation for routing open shop. Minimal routing relaxation for routing open shop.
6. Complexity and approximation for routing flow shop.

**Method of assessment**

In the program of the course, the examination is planned.

**Basic literature**

1. . P. Brucker. Scheduling algorithms. Springer-Verlag, Berlin, Germany, second, revised and enlarged edition, 1998.
2. . Ali Allahverdi, C.T. Ng, T.C.E. Cheng, Mikhail Y. Kovalyov. A survey of scheduling problems with setup times or costs. European Journal of Operational Research 187 (2008) 985–1032.
3. . G. Laporte. The Travelling Salesman Problem: An overview of exact and approximate algorithms. European Journal of Operational Research 59 (1992) 231-247.

**Discipline:**

**English Academic Writing**

**Discipline description**

Training on this discipline will be carried out in the form of special seminars under the chairmanship of leading researchers of ICM&MG SB RAS. Participation of leading members of editorial boards of “Siberian Journal of Numerical mathematics” (see the site <http://www.sscc.ru/>), “Siberian Mathematical Journal” (see the site <http://www.springer.com/mathematics/journal/11202>), journal “Monte Carlo Methods and Applications” (see the site <http://www.degruyter.com/view/j/mcma>) and others is also supposed. The discipline provides the student’s practice in analyzing and writing mathematical texts in English.

**Learning outcomes of the discipline**

As the result of discipline students gain skills for analyzing and writing mathematical texts in English (including texts on numerical statistical modelling and simulation).

**Method of assessment**

In the program of the course, the carrying out the test is provided.

**Basic literature**

Kutateladze S.S. Russian-English in writing. Novosibirsk: Institute of Mathematics SB RAS, 2000 [in Russian]

**Course:**

**History of Numerical Statistical Modeling and Simulation**

(course of the MEP «Numerical Statistical Modeling and Simulation. Monte Carlo Methods»)

**Author**:

**Anton Vaclavovich Voytishek**

- Full Professor, Doctor of physical and mathematical sciences;

- professor of CCM DMM NSU; see <http://mmfd.nsu.ru/mmf/kaf/cm/prep.asp>;

- leading researcher of LSP ICM&MG SB RAS; see <http://osmf.sscc.ru/mixa/bak.html>;

- see also the section “Information about the supervisor of the programme” of this programme;

- e-mail: [vav@osmf.sscc.ru](mailto:vav@osmf.sscc.ru)

**Course description**

The role of numerical statistical modelling and simulation (or Monte Carlo methods) increases with the development of computer technologies. Historically, the development of Monte Carlo methods was related to progress in mathematical modelling of nuclear processes (in order to produce corres-ponding technologies) in USSR and USA in fifties of the 20-th century.

Development beginning of the theory and applications of the Monte Carlo algorithms is connected with fundamental works of J.Neumann, S.Ulam, N.Metropolis, N.I.Buslenko, J.M.Hammmersley, J.Spanier, I.M.Sobol, S.M.Ermakov, G.A.Mikhailov, G.I.Marchuk, M.Kalos and members of their scientific schools.

Over the past half-century the applicability for methods of numerical statistical modelling and simulation greatly expended. The theory of probability representations for solutions of problems of mathematical physics was elaborated. On basis of this theory, the effective Monte Carlo estimates were constructed. Effective numerical algorithms were also elaborated for statistical physics (the Metropolis-Hastings algorithm, the Ising model), physical and chemical kinetics (multi-particle problems, solution of Boltzmann and Smoluchowski equations, modelling of reactions and phase transitions), queuing theory, financial mathematics, turbulence theory, mathematical biology, etc. The Monte Carlo algorithms allow effective parallelization for calculations on modern supercomputer equipment. In development of the mentioned above theories and numerical schemes, the significant (sometimes leading) role belongs to researches from Department of Statistical Modelling in Physics of Institute of Computational Mathematics and Mathematical Geophysics of Siberian Division of Russian Academy of Sciences (head – Corresponding Member of USSR Academy of Sciences, Professor G.A.Mikhailov).

In the course “History of numerical statistical modelling and simulation” main stages of development of the theory and applications of Monte Carlo numerical algorithms are reflected. Special place in this course takes the review of scientific achievements of Novosibirsk school of Monte Carlo methods.

**Learning outcomes of the course**

As a result of study of the course “History of numerical statistical modelling and simulation” the student acquires detailed knowledge on development of theory and applications of Monte Carlo methods. It allows him to choose more competently his training trajectory in frames of the MEP “Numerical Statistical Modelling and Simulation. Monte Carlo Methods” and also the further scientific and expert activity in this field.

**Course content**

1. Buffon’s needle experiment

2. E.Fermi’s Monte Carlo experiments in the field of neutron diffusion.

3. Monte Carlo experiments in the Los Alamos Scientific Laboratory. Manhattan Project.

4. Review of works of Von Neumann, S.Ulam, N.Metropolis in the field of Monte Carlo methods

5. Review of the paper *Metropolis N., Ulam S. The Monte Carlo method // Journal of American Statistical Association. 1949. V. 44, № 249. P. 335-341*.

6. The nuclear project in the Soviet Union

7. Review of early works of G.I.Marchuk, G.A.Mikhailov, S.M.Ermakov, I.M.Sobol, N.N.Chentsov, D.A.Frank-Kamenetskii, N.S.Bahvalov, etc. in the field of Monte Carlo methods

8. Review of the monography *Buslenko N.P., Golenko D.I., Sobol I.M., Sragovich, V.G., Shreider Ju.A. Method of Statistical Trials (Monte Carlo Method). Moscow: State Publishing House of Physical-Methematical Literature, 1962 [in Russian].*

9. Review of the monorgaphy *Hammmersley J.M, Handscomb D.C. Monte Carlo Methods. New York: Jonh Wiley and Sons, 1964.*

10. Review of the monorgaphy *Spanier J., Gelbard E. Monte Carlo Principles and Newtron Transport Problems. Addison-Wesley, Reading, 1969.*

11. On Moscow Monte Carlo scientific school. Review of the monorgaphy *Sobol I.M. Numerical Monte Carlo Methods. Moscow: Nauka, 1973 [in Russian].*

12. On Leningrad (St-Petersburg) Monte Carlo scientific school. Review of the monorgaphy Ermakov S.M. Monte Carlo Method and Related Issues. Moscow: Nauka, 1974 [in Russian]

13. On Novosibirsk Monte Carlo scientific school. Review of the monorgaphy *Marchuk G.I., Mikhailov G.A., Nazaraliev M.A., Darbinjan R.A., Kargin B.A., Elepov B.S. The Monte Carlo Methods in Atmospheric Optics. Heidelberg: Springer-Verlag, 1980.*

14. Prof. G.A.Mikhailov as the Novosibirsk Monte Carlo scientific school.

15. The outstanding role of the textbook *Ermakov S.M., Mikhailov G.A. Statistical Modelling. Moscow: Nauka, 1982 [in Russian]* for development of theory and applications of Monte Carlo methods.

16. Blossoming of the Novosibirsk Monte Carlo scientific school – eightieth years of the twentieth century. Review of the monorgaphy. Mikhailov G.A. Optimization of Weighted Monte Carlo Methods. Heidelberg: Springer-Verlag, 1992.

17. Survival of the Novosibirsk Monte Carlo scientific school – ninetieth years of the twentieth century.

18. Revival of the Novosibirsk Monte Carlo scientific school in 2000-2013.

19. Monte Carlo methods teaching in Novosibirsk State University.

20. Review of the textbook *Mikhailov G.A., Voytishek A.V. Numerical Stochastic Modelling. Monte Carlo Methods. Moscow: Academia, 2006 [in Russian]*.

**Method of assessment**

According to the program of the course, the test is planned.

**Basic literature**

Voytishek A.V. Foundations of the Monte Carlo Method. Novosibirsk, 2013 (electronic version).

**Course:**

**Applied Statistics**

(course of the MEP «Probability and Statistics»)

**Authors**:

**Alexander Ivanovich Sakhanenko**

- Full Professor, Doctor of physical and mathematical sciences;

- Head of the Laboratory of Probability and Mathematical Statistics at the Sobolev Institute of Mathematics SB RAS;

- e-mail: sakh@math.nsc.ru

**Natalia Isaakovna Chernova**

- Candidate (PhD) of physical and mathematical sciences;

- Associate professor of the Chair of Probability Theory and Mathematical Statistics of Department of Mechanics and Mathematics (DMM) of Novosibirsk State University (NSU);

- e-mail: cher@nsu.ru

**Course description**

Statistics is the mathematical study of the collection, organization, analysis, interpretation and presentation of randomness and variability of data. It is one of the few major disciplines in which the researcher's art and experience can have significant impact in such different fields as as biological sciences and medicine, finance and insurance, telecommunication and manufacturing, geosciences and climatology, as well as economics and education.

The course “Applied Statistics” is an elective course in the main (variable) section of professional cycle of disciplines of Master Educational Programme (MEP) “Probability and Statistics”. This course can also be used in MEP “Numerical Statistical Modelling and Simulation. Monte Carlo Methods”, “Mathematical and Computer Modelling in Mechanics”, “Modern Trends in Discrete Mathematics and Combinatorial Optimization” of Department of Mechanics and Mathematics NRU NSU. This course requires the undergraduate courses on Probability Theory and Mathematical Statistics as prerequisites.

*The aim of the course* is to allow students to deepen their statistical analysis knowledge, to provide their ability to solve practical problems in various work contexts. This course will provide an introduction to modern applied statistical methods. The course will cover, in particular, the following topics: methods of sampling data and experimental design, non-parametric statistical methods, correlation analysis, simple and multiple regression, principal component analysis, one-way and two-way analysis of variance, time series analysis.

The *objectives* of the course “Applied Statistics” are to provide students with a strong foundation in mathematical and statistical methodology, experience in its applications, a solid background in the use of statistical methods, and the skills to communicate the results of statistical analysis.

**Learning outcomes of the course**

As the result of studying the course “Applied Statistics” the student

*should understand* the role and place of statistical methods in applications;

*should understand* the basic statistical inference methods;

*should be able* to understand fundamental concepts in statistical analysis;

*should be able* to apply their knowledge to solve practical problems requiring the use of statistical methods and to choose appropriate and adequate ones.

**Course content**

1. Background. Sampling distributions. Point and interval estimation. Basic statistical hypotheses, tests of parametric hypotheses, goodness-of-fit tests.

2. Regression analysis. Regression models and the least squares estimation. Simple and multiple linear regression. Regression diagnostics. Confidence intervals and tests of parameters. Variable selection.

3. One-way and two-way analysis of variance.

4. Rank and order statistics and their properties. Nonparametric hypotheses testing. Wilcoxon signed-ranks test, Mann-Whitney U test, binomial sign test, Cochran Q test, Wilcoxon matched-pairs signed-ranks test, Kruskal-Wallis one-way analysis of variance, Friedman two-way analysis of variance, chi-squared goodness-of-fit test, chi-squared test for homogeneity and independence, tests of randomness, correlation tests.

6. Applied time series analysis. Regression techniques for modeling trends, smoothing techniques, autocorrelation, partial auto-correlation, moving average, Box-Jenkins models, forecasting, model selection and estimation.

**Method of assessment**

The final grade bases on the result of test. Course program contains sample test questions and problems.

**Main reading**

1. Encyclopedia of Statistical Sciences, 16 Volume Set, 2nd Edition. S.Kotz (Editor-in-Chief). Wiley-Interscience, 2006, 9420 pages.

2. E. L. Lehmann, and Joseph P. Romano. Testing statistical hypotheses. Springer Texts in Statistics: Springer, New York, 3rd ed., 2005.

3. Norman R. Draper, Harry Smith. *Applied regression analysis*. Wiley, New-York, 3rd ed., 1998, 706 pages.

**Course:**

**Financial Mathematics**

(course of the MEP «Probability and Statistics»)

**Author**:

**Evgeny Anatol’evich Baklanov**

- Candidate (PhD) of physical and mathematical sciences;

- Associate Professor of the CPTMS DMM NSU;

- e-mail: baklanov@mmf.nsu.ru.

**Course description**

Financial Mathematics is a branch of mathematics designed to analyze the financial structure, operating under conditions of uncertainty and find the most rational ways to manage financial institutions and facilities, taking into account such factors as time, risk, stochastic evolution, etc.

The main objective of financial mathematics is to provide an adequate assessment tools, taking into account the probabilistic nature of market conditions and the flow of payments from the instruments. Also methods of assessment of financial risks are based on the models of financial mathematics. In the stochastic financial mathematics it is necessary to define the criteria for risk assessment, including an adequate assessment of the financial instruments.

Financial markets are the main object of Financial Mathematics, based on such probabilistic and statistical disciplines as Stochastic Processes, Statistics of Stochastic Processes, Martingale Theory, Stochastic Analysis, etc.

The *goals* and *objectives* of the course:

• to give basic concepts and results of Stochastic Financial Mathematics;

• to give applications to a variety of estimates in a stochastic financial engineering;

• to develope the basic knowledge of stochastic processes in economics and finance;

• to acquaint with the stochastic analysis and calculations in models of financial markets, which operate under conditions of uncertainty;

• to acquaint with practical skills in the using of stochastic methods for the calculation of the corresponding continuous econometric models;

• to acquaint with the ability to interpret mathematical results for the prediction and explanation of economic effects and management of economic systems.

**Learning outcomes of the course**

As the result of study of the course “Financial Mathematics” the student

*should know*basic concepts and results of Stochastic Financial Mathematics*;*

*should know*applications to a variety of estimates in a stochastic financial engineering;

*should know* the stochastic analysis and calculations in models of financial markets, which operate under conditions of uncertainty;

*should possess*methods of the stochastic analysis for solving test and advanced problems of economics and finance.

**Course content**

The course consists of four chapters. The first chapter presents the basic concepts and tasks of Financial Mathematics. In the second chapter we study the stochastic pricing model. In particular, we will consider the classic options: European options, Asian options, «lookback» options. The third chapter is an introduction to Stochastic Calculus. In particular, in the third chapter we study Ito integral - the main tool of the theory of the Stochastic Calculus.

In the fourth chapter we examine the calculation of the cost of derivatives in the continuous case and investigate in detail the Black-Scholes model.

***1. The basic concepts and tasks of financial mathematics***

The subject of financial mathematics. Examples of contracts. Arbitration. Hedging. Optimal investment prices payment obligations. Conditional expectations: definition, existence and uniqueness, basic properties.

Martingales: discrete and continuous time. Martingales, submartingales, supermartingales. Definitions and basic properties. Examples. Doob decomposition. Stopping times. Upcrossings. Basic inequalities. Convergence theorems. Uniformly integrable martingales. The discrete version of the stochastic integral. (B, S)-market and investment portfolio. The condition of self-financing. Discounting. The first and second fundamental theorem of financial mathematics. Incomplete markets. Upper hedging price.

***2. Stochastic models for pricing***

The concept of no-arbitrage in the market. The pricing model of financial assets. Arbitrage pricing theory. The calculation of arbitrage (fair) value of classic options: European options, Asian options, «lookback» options. The problem of optimal investment: a martingale approach. American options. Supermartingale characterization of the cost. The optimal time of the execute of the American options.

***3. Introduction to stochastic calculus***

Brownian motion (Wiener process): definition and basic properties. The construction of the continuous Brownian motion. Properties of the trajectories. The quadratic variation of the Brownian motion. The distribution of the maximum of the Brownian motion. Geometric Brownian motion.

The construction of the Ito stochastic integral. Properties of the stochastic integral. Ito’s change of variables formula. Ito process. The quadratic variation of the Ito integral and Ito process. Brownian bridge. Representation of geometric Brownian motion as an Ito process. Stochastic differential equation. Weak and strong solutions.

***4. The calculation of the value of derivatives in the continuous case.***

The Black-Scholes model. The Black-Scholes equation and its solution.

The Black-Scholes formula as a limiting case of the discrete Cox-Ross-Rubinstein formula. The Black-Scholes formula as the solution of a stochastic differential equation with the boundary conditions for a European call and put options. Girsanov theorem for the Brownian motion. Risk-neutral measure and Girsanov theorem in the general case. The derivation of the Black-Scholes using the Girsanov theorem.

Asian type options, "barrier" options. «Lookback» options. American call and put options. Inequalities relating the options of European and American types.

**Method of assessment**

In the program of the course, the carrying out the examination is provided.

**Basic literature**

1. Shiryaev A. N. Probability. New York, NY: Springer-Verlag, 1995.
2. Shiryaev A. N. Stochastic financial mathematics, 2002.
3. Oksendal B. Stochastic Differential Equations, New York, NY: Springer-Verlag, 2003.
4. Baxter M. W., Rennie A J. O. Financial Calculus. An introduction to derivative pricing. Cambridge University Press, Cambridge 2001.
5. Brzeźniak Z., Zastawniak T. J. Basic Stochastic Processes: A Course Through Exercises. Springer, 1999.
6. Shreve S. Stochastic Calculus for Finance I, II. Springer, 2004.
7. Steele M. Stochastic Calculus and Financial Applications. Springer, 2001.

**Course:**

**Number Theory**

**Authors**:

**Evgenii Petrovich Vdovin**

- Associate professor, Doctor of physical and mathematical sciences;

- professor of Chair of algebra and mathematical logic (CAML) in DMM NSU; see <http://mmfd.nsu.ru/mmf/kaf/aml/aml-e.html>;

- deputy director of IM SB RAS; see <http://math.nsc.ru/str/dir.htm>;

- e-mail: [vdovin@math.nsc.ru](mailto:vdovin@math.nsc.ru)

**Pavel Sergeevich Kolesnikov**

- Doctor of physical and mathematical sciences;

- associate professor of CAML DMM NSU; see <http://mmfd.nsu.ru/mmf/kaf/aml/aml-e.html>;

- head of the Laboratory of ring theory of IM SB RAS; see <http://math.nsc.ru/LBRT/a1/>;

- e-mail: [pavelsk@math.nsc.ru](mailto:pavelsk@math.nsc.ru)

**Course description**

The course contains a series of classical topics in number theory which can be informally divided into three parts: algebraic and transcendental numbers, distribution of primes, and p-adic numbers and their applications. The first section is partially motivated by the famous problem of “squaring the circle” known since the ancient times. Being unsolvable by means of straightedge and compass, this problem had been inspiring the development of algebra and geometry for centuries until it was proved in 1882 that the number ** is transcendental. The course covers the definitions and basic properties of algebraic complex numbers and algebraic integers, the Liouville’s theory of Diophantine approximations, and complete proofs of the theorems by C. Hermite and F. Lindemann that the fundamental constants *e* and ** are transcendental.

In the second section, we study the foundations of the analytic number theory. Founded by P. Dirichlet and B. Riemann in the middle of XIX century, this theory remains one of the most important branches of contemporary mathematics. The main idea of the analytic number theory is to apply well-developed techniques of advanced complex analysis to the arithmetic of natural numbers. The course focuses on two famous problems of this kind: determine the asymptotic behavior of the ratio of primes in the series of natural numbers and prove that each arithmetic progression of the form *an* = *a* + *nd* with relatively prime *a* and *d* contains infinitely many primes. Solutions of these problems are given by the famous Prime Number Theorem and the Dirichlet Theorem, respectively. The course contains complete proofs of these statements based on the properties of the Riemann zeta-function, Dirichlet series, and Chebyshev functions.

The third section contains the basics of *p*-adic number theory which plays an important role in contemporary geometry and mathematical physics, as well as in the general theory of Diophantine equations. We observe the theory of fields equipped with a valuation, classify the valuations on the field of rational numbers, and state the construction of the field of *p*-adic numbers. Finally, we establish the principal properties of *p*-adic numbers and shortly review some of their applications.

Prerequisites assumed to the students taking the course include: elementary number theory (division algorithm, fundamental theorem of arithmetic, Euclidean algorithm), basic algebra (ring of polynomials, structure of finite abelian groups), advanced complex-valued calculus (convergence of functional series and of parametrized improper integrals, Cauchy integral theorem), and basic topology (definition of a topological space, convergence, and topological complements).

**Course content**

1. The field of algebraic numbers and the ring of algebraic integers

2. Diophantine approximation of a real number by rational numbers. The Dirichlet Theorem on rational approximation. An example of a transcendental number

3. The numbers  and are transcendental

4. The distribution of primes: statement and history. Chebyshev functions

5. Discrete convolution and the product of Dirichlet series for arithmetic functions. Mobius inversion formula

6. Euler identity. Riemann zeta-function in the half-plane Re(*z*)>1

7. The connection between the integral Chebyshev function and the logarithmic derivation of the Riemann function

8. The analytic extension for the Riemann function in the half-plane Re(*z*)>0. Zeros of the Riemann function in Re(*z*)1

9. Proof of the asymptotic law for the distribution of primes. An asymptotic formula for the  prime

10. Group of characters for a finite abelian group. Orthogonality relations

11. Dirichlet theorem for the number of primes in an arithmetic progression

12. Valuation fields. The classification of valuations on the field of rational numbers. Complement of a valuation field

13. The construction and properties of the ring of integer *p*-adic numbers and of the field of *p*-adic numbers

**Method of assessment**

According to the program of the course, the examination is planned.

**Basic literature**

Vdovin E.P., Kolesnikov P.S. Number Theory. Novosibirsk, 2013 (electronic version).

**Course:**

**Methods of Discrete Simulation**

(course of the MEP «Mathematical and Computer Modeling in Mechanics»)

**Authors**:

**Yuriy Nikolalevich Grigoriev**

- full Professor, doctor of physical and mathematical sciences;

- Professor of Chair of Mathematical modeling in DMM NSU; see <http://www.ict.nsc.ru/matmod/>;

- chief researcher in Institute of Computational Technologies SB RAS; see <http://www.ict.nsc.ru>.

**Vasiliy Nikolaevich Lapin**

- candidate of physical and mathematical sciences (PhD);

- senior tutor, of CMM DMM NSU; see <http://www.ict.nsc.ru/matmod/index.php?file=prepods>;

- researcher of ICT SB RAS; see <http://www.ict.nsc.ru/staffall.php>:

**Course description**

Corse gives an introduction in numerical method of discrete particles. Universal formal approach to particle-in-cell algorithms construction is described. Most popular models of discrete particles are investigated. Specific errors of method are analyzed. General scheme of particle-in-cell method applications to problems in gas dynamics, vorticity dynamic in incompressible and compressible fluids plasma physics rear gas dynamic is proposed.

For each application typical problem solution are considered and new ways of the method application are shown.

The course *goals* are *to give a conception* about numerical particle methods as an alternative of finite difference methods, *teach to construct* algorithms of particle-in-cell method based on mathematical problem formulation, to improve methods (increase methods efficiency and reduce typical errors), *to implement* the methods for parallel computers.

Course duration is one semester. Lections are planned within the course. Overall complexity of the course is 2 credits divided equally among the all forms of training.

**Learning outcomes of the course**

As the result of study of the course « Methods of discrete simulation » the student should

* *know* the basic of theory and applications of particle-in-cell methods;
* *be able* to build particle-in-cell method for particular problem ;
* *be able* to use universal blocks and specific procedures of the method and reduce typical method errors.

**Course content**

1. Particle-in-cell methods in mathematical modelling.
   1. General properties
   2. Some applications
2. Particle-in-cell methods.
   1. General scheme
   2. Model particles and theirs properties
   3. Errors of particles in cells schemes.
3. Method of particles in gas dynamics
   1. Equations of gas dynamic in divergent form
   2. Scheme Harlow
   3. Method of super particles (FLIC method)
4. Vortexes in cells methods
   1. Vorticity dynamic in plane flows
   2. Vortexes in cells methods for incompressible floes
   3. Vortexes in cells methods for compressible floes
5. Particle-in-cell methods in plasma mechanics
   1. Kinetic equation with self-consistent field
   2. General scheme and computational cycle
6. Statistical particle-in-cell methods
   1. Kinetic equations of rear gas
   2. Some approaches of Monte-Carlo methods
   3. Bird’s algorithm

**Method of assessment**

In the program of the course, scientific report writing and carrying out an examination are provided

**Basic literature**

Yu.N. Grigoryev, V.A.Vshivkov and M.P.Fedoruk. Numerical-Particle-in-Cell Methods - Theory and Applications. - Utrecht: VSP BV.-2002. P.250+viii, ISBN 90-6764-368-8.

**Course:**

**Numerical Methods**

(course of the MEP «Mathematical and Computer Modeling in Mechanics»)

**Authors**:

**Sergey Grigorievich Cherny**

- Full Professor, doctor of physical and mathematical sciences;

- professor of CMM DMM NSU; see <http://www.ict.nsc.ru/matmod/index.php?file=prepods>;

- head of laboratory of mathematical modeling of ICT SB RAS; see <http://www.ict.nsc.ru/staffall.php>:

**Vasiliy Nikolaevich Lapin**

- candidate of physical and mathematical sciences (PhD);

- senior tutor, of CMM DMM NSU; see <http://www.ict.nsc.ru/matmod/index.php?file=prepods>;

- researcher of ICT SB RAS; see <http://www.ict.nsc.ru/staffall.php>:

**Course description**

The course presents detailed descriptions for approaches to develop numerical methods for ordinary and partial differential equations, train students skills in creation, development, and realization of the methods. The course also familiarizes students with methodic of computational experiment, provides the experience in its performing. Lections, practical seminars, laboratory works, report writing and presentations are planned within the course.

The *aim* of the course “Numerical methods” is to teach students to use numerical methods to solve partial differential equations.

The corresponding *objectives* of the course “Numerical methods” are:

* to give to students knowledge of numerical methods theory, methodology of computational experiment.
* train theirs practical skills in development and implementation of numerical method;
* to teach students to investigate numerical methods in terms of their efficiency and to use these algorithms for numerical solution of actual applied problems;

When developing the course, the vast experience of the author in teaching at Mechanical-Mathematical Department of NRU NSU the disciplines concerned the theory and practice of numerical method and mathematical modelling, are used.

The course “Numerical methods” can be used in realization of the international master programmes “Probability and Statistics”, “Numerical Statistical Modelling and Simulation. Monte Carlo Methods” and «Modern Trends in Discrete Mathematics and Combinatorial Optimization» of DMM NRU NSU.

**Learning outcomes of the course**

As the result of study of the course “Numerical methods” the student

*should know* various types of numerical methods of partial differential equation solution, their properties and features.

*should be able to* know most popular finite difference and finite volume methods for problems with ordinary and partial difference equations.;

*should be able to* develop numerical algorithms, implement in computer code typical numerical methods, pose and perform computational experiment, analyze and present computation results.

**Course content**

1. Mathematical models and numerical experiment
2. Numerical methods for Cauchy problem for ordinary differential equation (ODE).
3. Numerical methods for boundary problems for ODE
4. Finite difference schemes for parabolic equations
5. Finite difference schemes for elliptic equations
6. Finite difference schemes for hyperbolic equations

**Method of assessment**

In the program of the course, the carrying out the examination, tests laboratory work and reports are provided.

**Basic literature**

1. *Khakimzyanov G.S., Cherny S.G.* Numerical methods: Part 1. Numerical methods for Cauchy problem for ODE // Textbook . – Novosibirsk, NSU. – 2003. – 160 p.
2. *Khakimzyanov G.S., Cherny S.G.* Numerical methods: Part 3. Numerical methods for boundary problems for ODE// Textbook . – Novosibirsk, NSU. – 2005. – 160 p.
3. *Khakimzyanov G.S., Cherny S.G.* Numerical methods: Part 3. Numerical methods for parabolic and elliptical equations problem solution // Textbook . – Novosibirsk, NSU. – 2008. – 160 p.
4. *Lebedev A.S., Cherny S.G.* Practices for numerical methods for partial differential equations // Novosibirsk, – NSU – 2000 – 136p.
5. *Samarskiy A.A., Gulin A.V.* Stability of difference schemes – Moscow, Scientific world, – 2003.

**Course:**

**Modern Methods of Computational Mathematics**

(course of the MEP «Mathematical and Computer Modeling in Mechanics»)

**Authors**:

**Gayaz Salimovich Khakimzyanov**

- Full Professor, doctor of physical and mathematical sciences;

- professor of MM DMM NSU; see <http://www.ict.nsc.ru/matmod>;

- leading researcher of ICT SD RAS; see <http://www.ict.nsc.ru/showpers.php?uid=37> .

**Galina Gennadievna Lazareva**

- Assistant Professor, PhD in Mathematical Modelling and Computational Technology in Science;

- senior lecturer of MM DMM NSU; see <http://www.ict.nsc.ru/matmod>;

- senior researcher of ICMaMG SD RAS; see <http://sscc.ru>;

- see also the section «Information about the supervisor of the programme» of this programme.

**Course description**

Discipline program “Modern methods of computational mathematics” is compiled in accordance with requirements to the structure and results of master educational programs “Professional cycle. Free electives”. It is an item of educational master program in English “Mathematical and Computer Modeling in Mechanics”. Discipline program is compiled in accordance with Novosibirsk State University tasks of implementing the NSU development Program.

The developed course “Modern methods of computational mathematics” belongs to the section “Basic part” and designed to study during the second year of the MEP “Mathematical and Computer Modeling in Mechanics”. When developing the course the possibility for control of various preparation levels of Russian and foreign students in the fields of probability theory, functional analysis and numerical statistical modelling and simulation is provided.

Note that the presented program is new. It is based on the latest achievements of recent years in the field of computational mathematics. A distinctive feature of the presented program is that the material of the lectures is not stated previously in textbooks and manuals. Lecture material is spread out over numerous journal articles. This creates difficulties both for undergraduates for self-study and development of modern tools of computational mathematics. Therefore, students will be useful and convenient to have on hand material. In its present and future research students have the knowledge contained the single style, give an idea of the current state of Affairs in computational mathematics.

Lecture course *aims to*  
• give the students basic knowledge in the field of the theory and applications of modern numerical methods;  
• teach students to choose the most effective numerical methods for solution of actual applied problems;  
• teach students to conduct modern level computing experiments.

The discipline consists of a set of four interconnected modules. The first one is connected with the modern finite-difference methods on adaptive grids. Students learn the concept of adaptive mesh, some methods of construction of adapting to the solution of one-dimensional nets and multidimensional curved grids. Students learn the General principles of constructing finite-difference schemes on adaptive grids. Special attention is paid to the development of schemes on adaptive grids for the Poisson equation, equations of shallow water and gas dynamics.

The second module focuses on the finite-element approximations. This course focuses on mastering the masters’ modern methods of triangulation areas with complex geometry borders.

In the third module outlines the features of such rapidly developing recently numerical methods as methods of control volumes of fictitious areas, boundary elements, spectral methods and various algorithmic method implementations «particles-in-cells». Maked an examples of problems solved by these methods.

The final module deals with some special issues of the modern computational mathematics. The theory TVD schemes, specific features of the development non-oscillating schemes for multidimensional problems are considered. Modern methods of monotonization of hyperbolic equations numerical solution and algorithms for accelerating the iterative convergence are studied.

**Learning outcomes of the course**

As a result of learning the course “Modern methods of computational mathematics” the student should:  
• Know techniques for constructing and theoretical methods of research of modern numerical methods;  
• Be award of the following skills: choice from a set of known numerical methods of the most effective numerical method for the particular class of applied problems;

• Know the modern technology of computing experiments.

**Course content**

1. Adaptive mesh refinement for solving second-order ordinary differential equations. The error of approximation on non-uniform mesh. General principles of constructing finite-difference schemes on adaptive grids.
2. Equidistribution method for building adaptive moving grids in the one-dimensional problems.
3. Predictor–corrector method on an non-uniform moving mesh for a one-dimensional linear transport equation. Schema properties.
4. The geometric conservation law and divergent schemes on the moving grid.
5. The concept of curvilinear net in a multidimensional region. Algebraic methods of construction of grids.
6. Differential methods of construction of adaptive meshes and their numerical realization.
7. Finite-difference scheme on an adaptive grid for the solution of a boundary-value problem for the Poisson equation. Integro- interpolation method of obtaining difference equations. Properties of differential operator.
8. Difference schemes on adaptive grids for shallow water equations. Schemes research.
9. Adaptive meshes in gas dynamics problems.
10. Finite element method for the second-order ordinary differential equations. Energy space. Generalized problem-solving.
11. Generalized solving of the Dirichlet problem for the Poisson equation. Finite element method for finding approximate generalized solution.
12. Domain triangulation. Methods of construction of unstructured triangular grids in the two-dimensional domains with complex geometry borders.
13. Control volume approach. Application.
14. Fictitious domains method. Hydrodynamic problems.
15. Boundary-element method. Free boundary problems.
16. Spectral methods. Meteorology problems.
17. Numerical methods «particles-in-cells». Wave hydrodynamics problems.
18. Theory TVD schemes. Theorem on necessary and sufficient condition of monotony of difference schemes with constant and variable coefficients.
19. Modern methods of monotonization for numerical solution of hyperbolic equations. Monotonization the predictor-corrector scheme for one-dimensional linear transport equation.
20. The design features of the non-oscillating schemes for multidimensional problems.
21. Modern algorithms for accelerating the iterative convergence. Algorithms for accelerating the iterative convergence using the least square technique.
22. Schemes of high order of approximation, based on the Runge-Kutta method. Multigrid schemes.
23. General principles of computing experiment organization. Mathematical model hierarchy and research methods. Computational algorithms hierarchy. Accuracy, stability, efficiency, parallelism numerical algorithms. Some principles of software engineering. Design, programming, debugging, software testing. Processing method of the results of calculations.

**Method of assessment**

In the program of the course, the carrying out the examination is provided.

**Basic literature**

Khakimzyanov G.S. Modern methods of computational mathematics. Novosibirsk, 2013 (electronic version).

**Course:**

**Theory of Programming**

(course of the MEP «Mathematical and Computer Modeling in Mechanics»)

**Authors**:

**Mikhail Alexeyevich Bul’onkov**

- Candidate of Physics and Math Sciences;

- Senior Lecturer of the Chair of Programming of the DMM NSU; see <http://programming.iis.nsk.su/>;

- Head of laboratory in IIS SD RAS; see http://www.iis.nsk.su:

**Natalya Nikolaevna Filatkina**

- Magister of Mathematics from NSU;

- Tutor of the Chair of Programming of the DMM NSU; see <http://programming.iis.nsk.su/>;

- Scientific researcher in IIS SD RAS; see <http://www.iis.nsk.su> .

**Course description**

The objective of the course is the student comprehension of basic concepts of theoretical programming, which in their turn are essential for understanding of modern technologies and methodology of software engineering and development. The purpose of the course “Theory of Programming” is to realize more in-depth learning of the section “Professional Cycle. Varying part (disciplines at the choice of the student)” of the MEP “Mathematical and Computer Modeling in Mechanics” and to study fundamental theory of program design and understanding. Notice that the course “Theory of Programming” corresponds to the obligatory discipline “Modern methods of computational mathematics” of the State Master Standard (SMS) for specialty 010800 – “Mechanics and mathematical modeling”.

The first part of the course is devoted to the elements of complexity theory as a mean for estimation of essential properties of computational models. This distinguishes the course from the course “Theory of Algorithms” that concentrates on the questions of existence of algorithms. Familiarity with various models and their comparative analysis on the basis of simulation gives the student the understanding of program implementation of a model and its complexity. The methods of estimation are demonstrated on the widely used search and sorting algorithms as well as the task of matrix-vector multiplication. The course provides a machine-independent complexity theory that allows to estimates the applicability limits of the concepts related to the complexity of algorithms.

The second part of the course considers a different aspect of program model analysis. The high level abstraction is leverage to uniform definition of various program characteristics, and dynamic transition systems in general, which allows applying the method of analysis to a wide range of problems. A student will obtain a conception of the very fundamental notions such as control and data flows, informational dependences, invariants, etc. The usefulness of these notions is demonstrated on the formulation of conditions for application of program transformations, which are used in a constructive proof of the decidability of logical-term equivalence.

The concluding part of the course addresses the theory of mixed computation – a generic method for improving program efficiency based on some knowledge of its input data. Many of the concepts, which were discussed earlier in the course, are shown to be useful for realization o mixed computations. On the other hand, mixed computation and its close relation to interpretation and compilation is a good link to implementation of programming languages. Finally, since the main purpose of mixed computation is program efficiency it utilizes many fundamental concepts of complexity theory.

The course “Theory of Programming” provides a systematic knowledge about the main directions of computer science. The course acquaints students with a wide range of problems arising in the scope of reliable and effective software development.

**Learning outcomes of the course**

As a result of learning the course “Theory of Programming” the student should:

* know basic concepts of algorithm complexity theory, program models, and mixed computation,
* be able to analyze complexity of algorithms and program models, and to apply mixed computation methods for compilation,
* be skilled in solving the problems related to algorithm design and program manipulation,
* have a solid reasoning for choice of adequate data structure and storage for a task being solved.

**Course content**

1. The concept of Turing machine (MT). The concept of RAM. Time and space complexity. The concept of complexity in average.
2. Simulation of MT on RAM and of RAM on MT. Complexity estimation.
3. Theorem of linear speed-up. The concept of complexity measure. Tseitin and Rabin theorems.
4. Lower bounds of complexity. Unbranching programs as a computation model. The theorems of lower complexity bounds of matrix-vector multiplication for the cases of rows and columns.
5. The concept of finite state machine (FSM). The concept of regular expression and its language. The theorem of equivalence of FSM and regular languages. Pumping lemma.
6. The problems of emptiness and equivalence for FSM. The concept of minimal automaton and its construction by the roughest splitting method.
7. Search in informational space. Bit scales: constant time inclusion and search. Linear search, time complexity estimation in the worst case and in average. Binary search and binary search trees. Balanced trees. 2-3-trees: complexity estimation, insertion and deletion. B-trees as a generalization of 2-3-trees: complexity estimation. AVL-trees. Hash tables.
8. Sorting. Classification of sorting methods. The concept of decision tree. Complexity estimation in the worst case or sorting based on comparison. Merge sort and quick sort: time complexity estimation. The concept of digital search.
9. The concept of marking algorithm. The formulation of global analysis problem (GAP). Exact solution of GAP, Non-deterministic marking process. Stable marking and the proof of its existence. Safe marking. The theorem of safe and unique stable marking. Estimation of complexity of GAP.
10. The theorem of existence of exact GAP solution for distributive case. Example of GAP for detecting loops in graphs. Methods of minimization of marking steps. The theorem of impossibility of construction of exaction GAP solution in monotonic case.
11. The concept of standard scheme: basis, terms, operators. The concept of interpretation: the value of term, protocol, results of computation. Free interpretations. Functional equivalence of standard schemes. Logical-term (LT) equivalence of standard schemes: LT-history, determinant. The theorem of LT-equivalence correctness.
12. The concept of informational relationships and informational graph. Construction of informational graph by formulation of GAP. Connected components of informational graph, and their interrelationship.
13. The concept of invariant equation. Functional nets as a representation of a set of invariant equations. Solution of detecting of invariant equation by GAP.
14. The concept of a system of transformations. Σlt system. Correctness of Σlt. Completeness of Σlt: matching of standard schemes, L-graphs. Estimation of complexity.
15. The concept of mixed computation (MC): programs as data, static and dynamic computation, basic MC equation. Relationship of MC with Kleene S-m-n theorem. Condition of expedience of MC. Relationship of MC, interpretation, and compilation of programs. Futamura projections. The concepts of meta-interpreter, self-interpreter, Jones optimality. Potential application of MC.
16. MC on the example of xn program. Interpretive approach to MC implementation. The concept of generating extension. Transformational approach.
17. The problem of dynamic control in MC. Polyvariant MC. Methods of program transformation to improve its MC.
18. The concept of binding time analysis (BTA). Reduction of BTA to GAP. Solution of BTA by fixed point iteration. Reduction of BTA to the solution of linear inequation system. The concept of polyvariant BTA. Transformation of program during its analysis.

**Method of assessment**

The course provides the following forms of training: lectures, seminars and control tests, and individual work as well. Accomplishing of program projects and testing results are taken into account to integrally estimate student’s course progress. The Faculty Academic Council can set additional forms of the control. Overall complexity of the course is 4 credits divided equally among the all forms of training.

**Basic literature**

1. Aho, Hopcroft, J., Ullman Data Structures and Algorithms: Trans. from English. - Moscow: Publishing House 'Williams', 2000.
2. Bulyonkov MA Mixed computation. Textbook. - Novosibirsk: Publishing House of the NSU, 1995.
3. Yuri Karpov Automata Theory. Textbook for high schools. - St. Petersburg.: Peter Publishing House, 2002.
4. T. Cormen, Charles E. Leiserson, Rivest R. Algorithms: construction and analysis: Per. from English. - Moscow: MCCME, 1999.
5. VE Kotov Introduction to the theory of program schemes. - Moscow: Nauka, 1978.
6. Kotov VE, VK Sabel'feld The theory of program schemes. - Moscow: Nauka, 1991.
7. S. Lavrov Programming. Mathematical foundations, funds theory. - St. Petersburg.: BHV-Petersburg, 2001.
8. Motwani, R. Hopcroft, J. Ullman, J. Introduction to Automata Theory, Languages, and Computation: Trans. with English, 2nd edition. - Moscow: Publishing House 'Williams', 2002.
9. Sabel'feld VK The theory of programming. Textbook. - Novosibirsk: Publishing House of the NSU, 1993.
10. Handbook of Theoretical Computer Science. Volume A: Algorithms and Com-plexity. Volume B: Formal Models and Semantics / Edited by J. van Leeuwen. - Cambridge, MA: The MIT Press and - Amsterdam, New York, Oxford, Tokyo: Elsevier, 1996.

**Course:**

**Stochastic Processes**

(course of the MEP «Probability and Statistics»)

**Author**:

**Pavel Sergeevich Ruzankin**

- Candidate of physical and mathematical sciences

- Associate Professor of the Chair of Probability Theory and Mathematical Statistics at the Department of Mechanics and Mathematics of Novosibirsk State University (NSU); see http://www.nsu.ru/mmf/tvims

- Senior Researcher of the Laboratory of Probability Theory and Mathematical Statistics at the Sobolev Institute of Mathematics of Siberian Branch of RAS.

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**Course description**

Original lecture course Stochastic Processes was developed for teaching foreign students studying Probability and Statistics in English. This is a compulsory course in the main (variable) section of professional cycle of disciplines of Master Educational Programme (MEP) “Probability and Statistics”. This course can also be used in MEP “Numerical Statistical Modelling and Simulation. Monte Carlo Methods”, “Mathematical and Computer Modelling in Mechanics”, “Modern Trends in Discrete Mathematics and Combinatorial Optimization” of Department of Mechanics and Mathematics NRU NSU. This course is based on the core courses on Probability Theory. As a prerequisite, a modest understanding of Probability theory at an undergraduate level is assumed.

The main part of the course is concerned with the theory and applications of stochastic processes. The widely-spread classes and representatives of stochastic processes are discussed. Certain limit theorems are given. Some attention is given to basic classes of queuing systems.

The aimof the course “Stochastic Processes” is to acquaint students with actual theoretical aspects of stochastic processes theory, to develop students' skills in the use of probabilistic methods for solving mathematical problems as a part of research on the theory of stochastic processes, and in other areas of mathematics. To achieve this goal the following course objectives are formulated: to introduce students to the basic concepts and methods of the theory of stochastic processes, to give an idea of the current state and development of the area, to form students' skills in conceptual apparatus of the theory of stochastic processes.

**Learning outcomes of the course**

As the result of study of the course “Stochastic Processes” the student

*should understand* the role and place of this discipline among the other mathematical subjects;

*should know* the definitions, basic properties and examples related to the Stochastic Processes and their practical value;

*should be able* to apply their knowledge to solve mathematical problems.

**Course content**

* Definition of a stochastic process. Distribution of a process. Sample probability space.
* Markov chains with discrete time. Classification of states. Ergodicity of Markov chains.
* Continuous-time Markov chains. Ergodicity. Kolmogorov’s differential equations. Generating matrix. Death and birth processes.
* Processes with independent increments.
* Poisson processes. Properties of trajectories.
* Kolmogorov’s theorem on existence of a continuous modification of a process.
* Wiener processes. Continuity of paths. Non-differentiability of paths. The law of iterated logarithm.
* Convergence of a sequence of Bernoulli processes to the corresponding Poisson process.
* The functional central limit theorem (the Donsker-Prokhorov invariance principle).
* Multi-dimensional Gaussian distributions, Gaussian processes.
* Poisson point processes (spatial Poisson processes). Modelling of such processes.
* Processes in *L*2-space of random variables. Continuity and differentiability of these processes.
* The Riemann integral for *L2*-processes.
* Elementary stochastic orthogonal measure. Stochastic integral of non-random function.
* Ito’s integral.
* Simple stochastic differential equations.
* M/M/1 queuing system. Convergence to the stationary distribution.
* M/G/1 queuing system. The stationary distribution of the embedded Markov chain. The waiting times. The busy period.
* G/M/1 queuing system. The stationary distribution of the embedded Markov chain. The waiting times.
* G/G/1 queuing system. The asymptotic waiting time. The embedded random walk.
* Closed migration processes. The equilibrium distribution.
* Open migration processes. The equilibrium distribution.

**Method of assessment**

The program of the course provides for the following types of discipline assessment: two control tasks and the colloquium, the interim test control and oral exam.

**Literature**

* Borovkov, A. A.: Probability Theory. Springer, 2013
* Rozanov, Yu. A.: Itroduction to the theory of random processes. Nauka, 1982.
* Korshunov, D. A., Voss, S.G., Eisymont, I. M.: Problems and exercises in probability theory. Lan, 2004.
* Grimmett, G., Stirzaker, D.: Probability and Random Processes. Oxford University Press, 2001.

**Course:**

**Numerical Modeling of Discrete Random Processes and Fields**

(course of the MEP «Numerical Statistical Modeling and Simulation. Monte Carlo Methods»)

**Author**:

**Vasilii Aleksandrovich Ogorodnikov**

- Associate Professor, Doctor of physical and mathematical sciences;

- professor of CCM DMM NSU; see <http://mmfd.nsu.ru/mmf/kaf/cm/prep.asp>;

- major researcher of LSP ICM&MG SB RAS; see <http://osmf.sscc.ru/mixa/bak.html>;

- see also the section «Information about the author» of working program of the discipline **M.2-B-5**;

- e-mail: [ova@osmf.sscc.ru](mailto:ova@osmf.sscc.ru)

**Course description**

Some questions connected with the construction and study of algorithms for simulation of Gaussian processes and fields of discrete argument based on the method of conditional distributions are considered in the given course. For the given Toeplitz and block-Toeplitz covariance matrices some special recursive algorithms for simulation of scalar and vector stationary processes and homogeneous fields on uniform grids are considered. Some questions of regularization and estimation the accuracy of the simulation are explored.

The application of these algorithms for scalar and vector autoregression process with a given correlation structure is considered, the conditions of their stationarity are studied. Special attention is paid to the modelling of non-Gaussian random processes and fields of discrete argument and also to the problems of numerical simulation of periodically correlated processes and conditional random fields.

The course “Numerical modelling of discrete random processes and fields” will meet high international level in the field of scientific research on the theory and applications of algorithms for discrete-stochastic numerical simulations.

The course is associated with the implementation of Development Program NRU NSU in 2009-2018 years and related to program sections “Discrete and Computational Mathematics”, “Modelling and analysis of the results of physical experiments”, “Technologies of distributed and high-performance computing and systems” of the direction “Mathematics, fundamental basis of computer science and information technology”.

**Learning outcomes of the course**

After studying of the course “Numerical modelling of discrete random processes and fields” a master student will know the methods and algorithms for the numerical modelling random Gaussian and non-Gaussian scalar and vector stationary and periodically correlated processes, homogeneous and homogeneous isotropic fields of discrete argument. A master student will also know the methods and algorithms for modelling the conditionally distributed Gaussian and non-Gaussian processes and fields of discrete argument, as well as methods of stochastic interpolation of random processes and fields.

**Course content**

1. Gaussian processes and fields of discrete argument with Toeplitz’s covariance matrixes

2. Method of conditional distributions for modelling normal vectors with Toeplitz’s correlation matrixes

3. Regularization of the algorithm, control of calculation accuracy

4. Modelling of autoregressive processes with the given correlation structure

5. Simulation of stationary Gaussian vector sequences with a given correlation structure

6. Simulation of homogeneous and homogeneous isotropic Gaussian fields on the regular grid

7. Modelling of stationary vector autoregressive sequences

8. Modelling of periodically correlated processes of discrete argument

9. Algorithms of modelling of non-Gaussian processes of discrete argument

10. Modelling of conditionally distributed Gaussian and non-Gaussian processes and fields of discrete argument

11. Combined models of non-Gaussian random processes and fields

12. Tasks, control questions

**Method of assessment**

In the program of the course, the carrying out an examination is provided.

**Basic literature**

Ogorodnikov V.A. Numerical Modelling of Discrete Random Processes and Fields. Novosibirsk, 2013 (electronic version).

**Course:**

**Random Walks**

(course of the MEP «Probability and Statistics»)

**Author**:

**Dmitry Alekseevich Korshunov**

- Doctor of physical and mathematical sciences

- Professor of the Chair of Probability Theory and Mathematical Statistics at the Department of Mechanics and Mathematics of Novosibirsk State University (NSU); see http://www.nsu.ru/mmf/tvims

- Leading Researcher of the Laboratory of Probability Theory and Mathematical Statistics at the Sobolev Institute of Mathematics of Siberian Branch of RAS; see http://math.nsc.ru/LBRT/v1/dima/dima.html

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**Course description**

Original lecture course Random Walks was developed for teaching foreign students studying Probability and Statistics in English. This is a basic course in the main (variable) section of professional cycle of disciplines of Master Educational Programme (MEP) “Probability and Statistics”. This course can also be used in MEP “Numerical Statistical Modelling and Simulation. Monte Carlo Methods”, “Mathematical and Computer Modelling in Mechanics”, “Modern Trends in Discrete Mathematics and Combinatorial Optimization” of Department of Mechanics and Mathematics NRU NSU. This course is based on the core courses on Probability Theory and Stochastic Processes. As a prerequisite, a modest understanding of Probability theory at an undergraduate level and of some basic notions in Stochastic Processes is assumed.

We present various techniques for proving limit theorems for random walks, such as method of characteristic functions, method of moments and Stein's method. We prove both integral and local limit theorems and key renewal theorem as well. Connections with applications from queueing theory and risk theory are discussed. The large deviation theory for the supremum of a random walk is presented as well, for both light-tailed and heavy-tailed distributions.

The aimof the course “Random Walks” is to acquaint students with actual theoretical aspects of random walks theory, to provide student with the knowledge and ability to apply modern methods of research in the analysis of random walks arising in applications.

The *objectives* of the course “Random Walks” are to provide students with advanced knowledge of basic concepts in the subject matter, to supply students by sound knowledge of modern methods of research in the area, to train student's ability to identify, summarize, formulate and solve theoretical problems in certain areas of mathematics.

**Learning outcomes of the course**

As the result of study of the course “Random Walks” the student

*should understand* the role and place of this discipline among the other mathematical subjects;

*should know* the definitions, basic properties and examples of random walks and their practical value;

*should be able* to apply their knowledge to solve mathematical problems.

**Course content**

Ch a p t e r I. **Limit theorems for random walks**

1. Some basic results from Probability Theory

2. Examples of random walks

3. Single server queueing system

4. Ruin Probabilities in Cramer-Lundberg Model

5. Characteristic functions and their properties

6. Inversion formulas for characteristic functions

7. Theorem of continuity for characteristic functions

8. Central limit theorem

9. Local limit theorem in lattice case

10. Local limit theorem for densities

11. Local limit theorem for intervals

12. On the recurrence of integer valued random walk

Ch a p t e r II. **Renewal theory**

13. Denitions

14. Law of large numbers for the renewal process

15. Central limit theorem for the renewal process

16. Integral renewal theorem

17. Key renewal theorem

18. Renewal theory on the whole real line

Ch a p t e r III. **Subexponential distributions**

19. Heavy-tailed and light-tailed distributions

20. Long-tailed functions and their properties

21. Long-tailed distributions

22. Long-tailed distributions and integrated tails

23. Subexponential distributions on the positive half-line

24. Subexponential distributions on the whole real line

25. Subexponentiality and weak tail-equivalence

26. Strong subexponential distributions

27. Kesten's bound

28. Subexponentiality and randomly stopped sums

Ch a p t e r IV. **Ladder structure of random walk**

29. Denition of ladder heights and epochs

30. Taboo renewal measures

31. Asymptotics for the first ascending ladder height

Ch a p t e r V. **Maximum of random walk**

32. Asymptotics for the maximum of a random walk with a negative drift in heavy-tailed case

33. Cramer-Lundberg approximation in light-tailed case

34. Explicitly calculable ascending ladder heights

35. Single server queueing system

36. Ruin probabilities in Cramer-Lundberg model

**Method of assessment**

Oral exam.

**Literature**

1. Asmussen, S.: Applied Probability and Queues, 2nd Edn. Springer, New York (2003)  
2. Asmussen, S.: Ruin Probabilities. World Scientific, Singapore (2000)  
3. Feller, W.: An Introduction to Probability Theory and Its Applications, vol. 2. Wiley, New York (1971)  
4. Foss, S., Korshunov, D., Zachary, S.: An Introduction to Heavy-Tailed and Subexponential Distributions. Springer (2013)

5. Korshunov, D.: Random Walks. Novosibirsk, NSU (2013, electronic version)

6. Rolski, T., Schmidli, H., Schmidt, V., Teugels, J.: Stochastic Processes for Insurance and Finance. Wiley, Chichester (1998)

**Course:**

**Markov Chains**

(course of the MEP «Probability and Statistics»)

**Author**:

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**Course description**

The course “Markov Chains” is an elective course in the main (variable) section of professional cycle of disciplines of Master Educational Programme (MEP) “Probability and Statistics”. This course can also be used in MEP “Numerical Statistical Modelling and Simulation. Monte Carlo Methods”, “Mathematical and Computer Modelling in Mechanics”, “Modern Trends in Discrete Mathematics and Combinatorial Optimization” of Department of Mechanics and Mathematics NRU NSU. As a prerequisite, a modest understanding of Probability theory at an undergraduate level and of some basic notions in Stochastic Processes is assumed. The course substantially deals with discrete-time Markov models both in discrete state space and in general state space.

The course contains main notations and definitions, introduce class structure for Markov Chains with discrete state space, discusses hitting times and absorption probabilities, recurrence and transience and limit theorems on the conditions of convergence to an equilibrium distribution for the chains with discrete state space. For Markov Chains in general state spaces, the various threads are possible. The strand chosen by this course mainly follows selected topics of famous S.P. Meyn and R.L. Tweedie book “Markov Chains and Stochastic Stability” and looks at the Markov chains in general state spaces through the prism of regeneration.

The aimof the course “Markov Chains” is to acquaint students with actual theoretical aspects of Markov chains theory, to provide student with the knowledge and ability to apply modern methods of research in the analysis of Markov models arising in applications.

The *objectives* of the course “Markov Chains” are to provide students with advanced knowledge of basic concepts in the subject matter, to supply students by sound knowledge of modern methods of research in the area, to train student's ability to identify, summarize, formulate and solve theoretical problems in certain areas of mathematics.

**Learning outcomes of the course**

As the result of studying the course “Markov Chain” the student

*should understand* the role and place of this discipline among the other mathematical subjects;

*should know* the definitions, basic properties and examples of Markov chains and their practical value;

*should be able* to apply their knowledge to solve mathematical problems.

**Course content**

1. Countable Markov chains. Definition and basic properties. Examples. Time-homogeneity. Random times and strong Markov property. Class structure. Hitting times. Recurrence and transience. Invariant distributions. Limiting behavior of recurrent Markov chains. Transformations of Markov chains.

2. Harris Markov chains. Basic definitions. Occupation, hitting and stopping times. Irreducibility. Irreducible models on a countable space

. Irreducibility on general state space. Minorization condition. Small sets. Harris recurrence. Existence of invariant measure. Ergodicity. Geometric ergodicity.

**Method of assessment**

The final grade is based on the results of oral exam. Course program contains sample exam questions and the detailed grade for this exam.

**Main reading**

1. J. G. Kemeny and J. L. Snell. Finite Markov Chains. Van Nostrand, Princeton, N.J. 1960.

2. O.Hernández-Lerma, *J.B. Lasserre*. Markov Chains and Invariant Probabilities. Birkhauser Verlag AG, 2003.

3. S. P. Meyn and R.L. Tweedie. Markov Chains and Stochastic Stability. Cambridge University Press, 2009.

**Discipline:**

**Scientific Seminars**

Note that participating in scientific seminars is one of the most important elements of famous Russian research schools. During scientific seminars a student can listen talks of other specialists in his field and deliver his own talks. Usually, making one talk during the semester and visiting most meetings is sufficient for the assessments. The Chair of Theoretical Cybernetics has the following scientific seminars:

Factor languages

Coding theory

Mathematical models of decision making

Discrete extremal problems

Discrete analysis

Introduction to discrete mathematics

Graph Theory

Cryptography and cryptanalysis

**APPENDIX 1. List of competencies of the graduates of the MEP**

According to the SMS for specialty 010200 – «Mathematics and Computer Science» (details [in Russian] can be found at the site <http://www.referent.ru/1/150164>), graduates of the MEP «Modern Trends in Discrete Mathematics and Combinatorial Optimization» should possess the following **general cultural competences (GCC)**.

**GCC-1:** Ability to work in an interdisciplinary team

**GCC-2:** Ability to cooperate with specialists from other fields

**GCC-3:** Ability to work in an international surrounding

**GCC-4:** ability to use advanced knowledge of rules of law and ethical standards in the evaluation of own professional activities, and also in the development and implementation of social projects.

**GCC-5: A**bility to generate new ideas and apply in research and professional practice the basic knowledge of fundamental and applied mathematics and natural science

**GCC-6:** Significant skills of independent scientific and research work and also work as a part of a team

**GCC-7:** Ability to improve and develop the intellectual level, initiative and aspiration for leadership

**GCC-8:** Ability to adapt fast to any situations

**GCC-9:** Ability to plan own work and the work of a team

**GCC-10:** Ability to find, analyze, and adequately process information related to technical science, natural science, and general science, and transform it into a form convenient for solving current problems.

According to the SMS for specialty 010200 – «Mathematics and Computer Science», graduates of the MEP «Modern Trends in Discrete Mathematics and Combinatorial Optimization» should possess the following **professional competences (PC)**.

**PC-1:** Ability to apply methods of mathematical modeling for analysis of global problems on the base of the deep knowledge of mathematics and computer science.

**PC-2:** Ability to apply methods of mathematical and algorithmic modeling for analysis of natural science problems.

**PC-3:** Ability to carry out research and to acquire new scientific and applied results.

**PC-4:** Independent analysis of physical aspects of classical mathematical problems.

**PC-5:** Ability to present own new scientific results.

**PC-6:** Ability to understand a scientific course completely and independently.

**PC-7:** Understanding of modern algorithms of the computer science and ability to improve and develop the mathematical theory lying in a base of such algorithms

**PC-8:** Ability to see applications of abstract mathematical formulations

**PC-9:** Ability to creatively develop and apply complicated mathematical algorithms in modern program complexes.

**PC-10:** Ability to determine general forms, regularities and instrumental tools for groups of disciplines.

**PC-11:** Ability to apply methods of mathematical and algorithmic modeling for analysis of economics and social processes, financial and business problems.

**PC-12:** Ability to adapt mathematical terminology and presentation to the level of the audience.

**PC-13:** Ability to supervise the research teams work.

**PC-14:** Ability to formulate non-mathematical types of data (including humanitarian) in the form of mathematical problems

**PC-15:** Ability to teach mathematics and computer science in the schools of all levels and in universities on the base of obtained fundamental education and scientific ideology

**PC-16:** Ability to obtain scientific information from electronic libraries and abstract journals

**APPENDIX 2. Examples of interview tests and control questions for the MEP**

**«Modern Trends in Discrete Mathematics and Combinatorial Optimization»**

**Test tasks examples**

1. Find the limit: *.*

2. Calculate the determinant of the matrix



3. Find the semiaxis lengths for the intersection curve of the surface** and the plane *.*

4. Let **  for ** and ** for **. Is the function ** continuous in the point **? Is it differentiable in the point **? Answers must be explained.

5. Proof the inequality: ** for all *.*

6. Find the minimum of the function 5*x*1 – *x*2 + *x*3 – *x*4 under the restrictions

– *x*1 + 2*x*2 + *x*3 = 2

*x*1 + *x*2 + *x*4 = 2 *xi*≥0, *i*=1,2,3,4

7. Among 100 cats 12 are black. Find a probability for exactly 2 among 10 randomly chosen cats being black.

**Examples of interview control questions**

1. What is the real number?

2. What is the limit of a sequence?

3. What is the limit of function in a point?

4. Define the continuous function.

5. Give the definition of the function’s derivative.

6. Give the definition of smooth function.

7. Give the definition of the Riemann integral.

8. Give the definition of the Lebesgue integral.

9. What is the matrix?

10. What is the inverse matrix?

11. What is the spectral value of a matrix?

12. Define the difference numerical approximation of a function’s derivative.

13. What is the quadrature formula?

14. What is the cubature formula?

15. What is the Monte Carlo method?

16. What is the discrete random variable?

17. What is the continuous random variable?

18. What is the mathematical expectation of a random variable?

19. What is the variance (dispersion) of a random variable?

20. What probabilistic distributions do you know?

21. Formulate the big numbers law.

22. Formulate the central limit theorem.

23. What is a Turing machine?

24. What programming languages can you use?

25. What kinds of modern supercomputing systems do you know?